

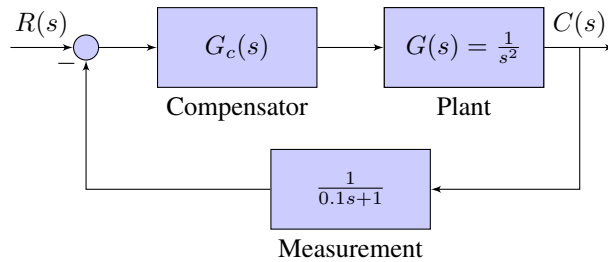
## Assignment: Compensator Design in Time Domain

### Problems

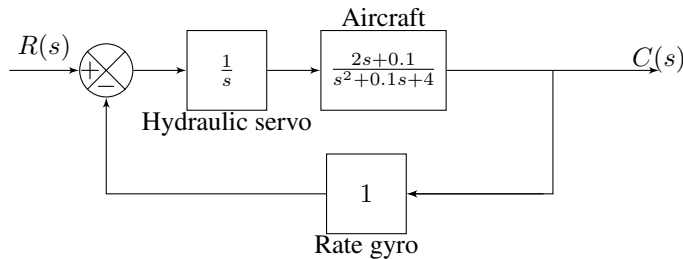
1. Consider a unity feedback system with open loop TF,  $G(s) = \frac{K}{(s+4)^3}$ .
  - (a) Find the location of the dominant poles to obtain settling time and overshoot as 1.6 second and 25% respectively.
  - (b) If a compensator with a zero at 1 is used to achieve the conditions of Part a, what must the angular contribution of the compensator pole be?
  - (c) Find the location of the compensator pole.
  - (d) Find the gain required to meet the requirements stated in Part a.
  - (e) Find the location of other closed-loop poles for the compensated system.
  - (f) Discuss the validity of your second-order approximation.
2. Consider a unity feedback system with open loop TF,  $G(s) = \frac{K(s+6)}{(s+3)(s+4)(s+7)(s+9)}$ 
  - (a) Plot the root locus.
  - (b) Find the coordinates of the dominant poles for  $\zeta = 0.8$ . Also determine gain for such poles.
  - (c) Now the system is compensated so that  $T_s = 1$ sec, and  $\zeta = 0.8$ . Find the compensator pole if the compensator zero is at -4.5.
  - (d) Discuss the validity of your second-order approximation.
3. Consider a unity feedback system with open loop TF,  $G(s) = \frac{K}{s(s+20)(s+50)}$ 
  - (a) The system is operating at 20% overshoot. Design a compensator to decrease the settling time by a factor of 2 without affecting the percent overshoot.
  - (b) Evaluate the uncompensated system's dominant poles, gain, and settling time.
  - (c) Evaluate the compensated system's dominant poles and settling time.
  - (d) Evaluate the compensator's pole and zero. Find the required gain.
4. Consider a unity feedback system with open loop TF  $G(s) = \frac{K}{s(s+1)(s^2+10s+26)}$ 
  - (a) Find the settling time for the system if it is operating with 15% overshoot.
  - (b) Find the zero of the compensator and the gain, K, so that the settling time is 7 sec. Assume that the pole of the compensator is located at -15.
5. Consider a unity feedback system with open loop TF  $G(s) = \frac{K}{(s^2+20s+101)(s+20)}$ 
  - (a) Design a suitable compensator such that the damping ratio for the dominant poles is to be 0.4, and the settling time is to be 0.5 second. Assume that the compensator pole is located at -15.
  - (b) Compare the performance of the uncompensated and compensated systems.

6. Consider a unity feedback system with open loop TF,  $G(s) = \frac{K}{(s+3)(s+6)}$
- Design a lead compensator so that the system operates with a settling time of  $2/3$  second and a percent overshoot of 1.5.
7. Consider a unity feedback system with open loop TF  $G(s) = \frac{K}{s(s+1)(s+3)}$
- Design a compensator to yield settling time = 2.86 seconds; percent overshoot = 4.32%; the steady-state error is to be improved by a factor of 2 over the uncompensated system.
  - Compare the transient and steady-state error specifications of the uncompensated and compensated systems.
  - Compare the gains of the uncompensated and compensated systems.
  - Discuss the validity of your second-order approximation.
8. Consider a unity feedback system with open loop TF,  $G(s) = \frac{10}{s(s+4)}$ . Design a lag compensator  $G_c(s)$  such that the static velocity error constant  $K_v$  is  $50 \text{sec}^{-1}$  without appreciably changing the location of the original closed-loop poles. Draw (i) Root locus for compensated and uncompensated systems; (ii) Step and ramp response of the closed loop uncompensated and compensated system. Comments on the results.
9. Consider a unity feedback system with open loop TF,  $G(s) = \frac{10}{s(s+2)(s+8)}$ . Design a compensator such that the dominant closed loop poles are located at  $s = -2 \pm j2\sqrt{3}$  and the static velocity error constant  $K_v$  is equal to  $80 \text{sec}^{-1}$ .
10. Consider a unity feedback system with open loop TF,  $G(s) = \frac{K}{s(s+5)(s+11)}$ .
- Find the gain,  $K$ , for the uncompensated system to operate with 30% overshoot.
  - Find the peak time and  $K_v$  for the uncompensated system.
  - Design a lag-lead compensator to decrease the peak time by a factor of 2, decrease the percent overshoot by a factor of 2, and improve the steady-state error by a factor of 30.
11. Consider a unity feedback system with open loop TF,  $G(s) = \frac{K}{(s+2)(s+4)}$ . The system is operated with 4.32% overshoot. In order to improve the steady state error,  $K_p$  is to be increased by at least a factor of 5. A lag compensator of the form  $G(s) = \frac{s+0.5}{s+0.1}$  is to be used.
- Find the gain required for both the compensated and the uncompensated systems.
  - Find the value of  $K_p$  for both the compensated and the uncompensated systems.
  - Estimate the percent overshoot and settling time for both the compensated and the uncompensated systems.
  - Discuss the validity of the second-order approximation used for your results in Part c.
  - Plot the response for the uncompensated and compensated systems. What do you notice about the compensated system's response?

12. Consider a unity feedback system with open loop TF,  $G(s) = \frac{1}{s(s+1)(s+5)}$ . Design a laglead compensator such that the static velocity error constant  $K_v$  is  $50 \text{ sec}^{-1}$  and the damping ratio  $\zeta$  of the dominant closedloop poles is 0.5. (Choose the zero of the lead portion of the laglead compensator to cancel the pole at  $s = -1$  of the plant.) Determine all closed-loop poles of the compensated system. Draw (i) Root locus for compensated and uncompensated systems; (ii) Step and ramp response of the closed loop uncompensated and compensated system. Comments on the results.
13. Consider the model for a space-vehicle control system shown in Figure below. Design a lead compensator  $G_c(s)$  such that the damping ratio  $\zeta$  and the undamped natural frequency  $\omega_n$  of the dominant closed-loop poles are 0.5 and 2 rad/sec, respectively. Draw (i) Root locus for compensated and uncompensated systems; (ii) Step and ramp response of the closed loop uncompensated and compensated system. Comments on the results.



14. (a) Determine the closed-loop transfer function of this system.  
 (b) Plot the the unit-step response of this system. Why the response is highly oscillatory? What is the settling time?  
 (c) It is desired to speed up the response and also eliminate the oscillatory mode at the beginning of the response. Design a suitable compensator such that the dominant closed-loop poles are at  $s = -2 \pm j2\sqrt{3}$ ;

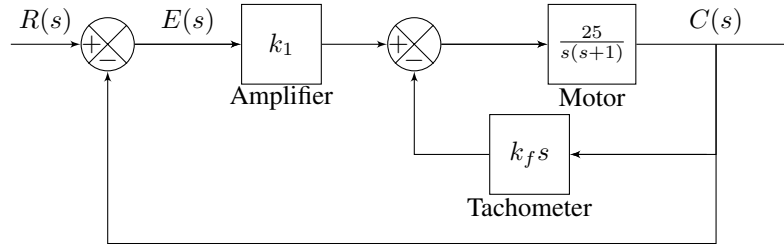


15. You are given the motor whose transfer is given by

$$E_a(s) \rightarrow \left[ \frac{25}{s(s+1)} \right] \rightarrow \theta_0(s)$$

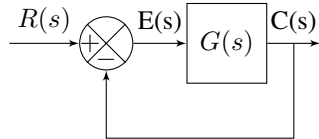
- (a) If this motor were the forward transfer function of a unity feedback system, calculate the percent overshoot and settling time that could be expected.

- (b) You want to improve the closed-loop response. Since the motor constants cannot be changed and you cannot use a different motor, an amplifier and tachometer are inserted into the loop as shown in Figure below. Find the values of  $k_1$ , and  $k_f$  to yield a percent overshoot of 25% and a settling time of 0.2 second.



- (c) Evaluate the steady-state error specifications for both the uncompensated and the compensated systems.

16. An X-4 quadrotor flyer is designed as a small-sized unmanned autonomous vehicle (UAV) that flies mainly indoors and can help in search and reconnaissance missions. To minimize mechanical problems and for simplicity, this aircraft uses fixed pitch rotors with specially designed blades. Therefore, for thrust it is necessary to add a fifth propeller. A simplified design of the thrust control design can be modeled as in Figure below with  $G(s) = G_c(s)P(s)$ , where  $P(s) = \frac{1.90978(s/0.43+1)}{(s/9.6+1)(s/0.54+1)}$  represents the dynamics of the thruster rotor gain, the motor, and the battery dynamics. Initially, the system is designed using a proportional compensator given by  $G_c(s) = 3$ .



- (a) Calculate the resulting steady-state error for a unit step input.  
 (b) Design a lag compensator to yield half the steady-state error of the proportional compensator, without appreciably affecting the system's transient response.  
 (c) Plot step response of compensated and uncompensated system.
17. Given the system in Figure below, find the values of  $k$  and  $k_f$  so that the closed-loop system will have a 4.32% overshoot and the minor loop will have a damping ratio of 0.8. Compare the expected performance of the system without tachometer compensation to the expected performance with tachometer compensation.

