

Test Paper Code : ST

Time : 3 Hours Max. Marks : 300

INSTRUCTIONS

- The question-cum-answer book has 40 pages and has 29 questions. Please ensure that the copy of the question-cum-answer book you have received contains all the questions.
- Write your **Roll Number, Name and the name of the Test Centre** in the appropriate space provided on the right side.
- Write the answers to the objective questions against each Question No. in the **Answer Table for Objective Questions**, provided on page No. 9. Do not write anything else on this page.
- Each objective question has 4 choices for its answer: (A), (B), (C) and (D). Only **ONE** of them is the correct answer. There will be **negative marking** for wrong answers to objective questions. The following marking scheme for objective questions shall be used :
 - For each objective question, you will be awarded **6 (six)** marks if you have written only the correct answer.
 - In case you have not written any answer for a question you will be awarded **0 (zero)** mark for that question.
 - In all other cases, you will be awarded **-2 (minus two)** marks for the question.
- Answer the subjective question only in the space provided after each question.
- Do not write more than one answer for the same question. In case you attempt a subjective question more than once, please cancel the answer(s) you consider wrong. Otherwise, the answer appearing later only will be evaluated.
- All answers must be written in blue/black/blue-black ink only. Sketch pen, pencil or ink of any other colour should not be used.
- All rough work should be done in the space provided and scored out finally.
- No supplementary sheets will be provided to the candidates.
- Logarithmic Tables / Calculator of any kind / cellular phone / pager / electronic gadgets are not allowed.**
- The question-cum-answer book must be returned in its entirety to the Invigilator before leaving the examination hall. Do not remove any page from this book.
- Refer to special instruction/useful data on the reverse.

READ THE INSTRUCTIONS ON THE LEFT SIDE OF THIS PAGE CAREFULLY

ROLL NUMBER				
Name :				
Test Centre :				

Do not write your Roll Number or Name anywhere else in this question-cum-answer book.

I have read all the instructions and shall abide by them.

.....
Signature of the Candidate

I have verified the information filled by the Candidate above.

.....
Signature of the Invigilator

NOTE: Attempt ALL the 29 questions. Questions 1 – 15 (objective questions) carry six marks each and questions 16 – 29 (subjective questions) carry fifteen marks each.

Write the answers to the objective questions ONLY in the *Answer Table for Objective Questions* provided on page 9.

1. Let

$$P = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 1 \\ 2 & 3 & 4 & 8 & 6 & 3 \\ 2 & 4 & 6 & 7 & 10 & 3 \\ 4 & 7 & 10 & 14 & 16 & 7 \end{bmatrix}$$

Then the rank of the matrix P is

- (A) 1
- (B) 2
- (C) 3
- (D) 4

2. Consider the following system of linear equations:

$$x + y + z = 3,$$

$$x + az = b,$$

$$y + 2z = 3.$$

This system has infinite number of solutions if

- (A) $a = -1, b = 0$
- (B) $a = 1, b = 2$
- (C) $a = 0, b = 1$
- (D) $a = -1, b = 1$

3. Six identical fair dice are thrown independently. Let S denote the number of dice showing even numbers on their upper faces. Then the variance of the random variable S is

(A) $\frac{1}{2}$

(B) 1

(C) $\frac{3}{2}$

(D) 3

4. Let X_1, X_2, \dots, X_{21} be a random sample from a distribution having the variance 5. Let

$$\bar{X} = \frac{1}{21} \sum_{i=1}^{21} X_i \quad \text{and} \quad S = \sum_{i=1}^{21} (X_i - \bar{X})^2.$$

Then the value of $E(S)$ is

(A) 5

(B) 100

(C) 0.25

(D) 105

5. Let X and Y be independent standard normal random variables. Then the distribution of

$$U = \left(\frac{X - Y}{X + Y} \right)^2 \text{ is}$$

- (A) chi-square with 2 degrees of freedom
- (B) chi-square with 1 degree of freedom
- (C) F with (2,2) degrees of freedom
- (D) F with (1,1) degrees of freedom

6. In three independent throws of a fair dice, let X denote the number of upper faces showing six. Then the value of $E(3 - X)^2$ is

- (A) $\frac{20}{3}$
- (B) $\frac{2}{3}$
- (C) $\frac{5}{2}$
- (D) $\frac{5}{12}$

7. Let

$$P = \begin{bmatrix} 1 & 0 & 1+x & 1+x \\ 0 & 1 & 1 & 1 \\ 1 & 1+x & 0 & 1+x \\ 1 & 1+x & 1+x & 0 \end{bmatrix}$$

Then the determinant of the matrix P is

- (A) $3(x+1)^3$
- (B) $3(x+1)^2$
- (C) $3(x+1)$
- (D) $(x+1)(2x+3)$

8. The area of the region $\{(x, y) : 0 \leq x, y \leq 1, \frac{3}{4} \leq x+y \leq \frac{3}{2}\}$ is

- (A) $\frac{9}{16}$
- (B) $\frac{7}{16}$
- (C) $\frac{13}{32}$
- (D) $\frac{19}{32}$

9. Let E, F and G be three events such that the events E and F are mutually exclusive, $P(E \cup F) = 1$, $P(E \cap G) = \frac{1}{4}$ and $P(G) = \frac{7}{12}$. Then $P(F \cap G)$ equals

- (A) $\frac{1}{12}$
(B) $\frac{1}{4}$
(C) $\frac{5}{12}$
(D) $\frac{1}{3}$

10. Let X and Y have the joint probability mass function

$$P(X = x, Y = y) = \frac{1}{3x}, \quad y = 1, 2, \dots, x; \quad x = 1, 2, 3.$$

Then the value of the conditional expectation $E(Y | X = 3)$ is

- (A) 1
(B) 2
(C) 1.5
(D) 2.5

11. Let X_1 and X_2 be independent random variables with respective moment generating functions

$$M_1(t) = \left(\frac{3}{4} + \frac{1}{4}e^t\right)^3 \text{ and } M_2(t) = e^{2(e^t - 1)}, \quad -\infty < t < \infty$$

Then the value of $P(X_1 + X_2 = 1)$ is

- (A) $\frac{81}{64}e^{-2}$
 (B) $\frac{27}{64}e^{-2}$
 (C) $\frac{11}{64}e^{-2}$
 (D) $\frac{27}{32}e^{-2}$

12. $\lim_{n \rightarrow \infty} \left[\frac{1}{2^{n/2} \Gamma\left(\frac{n}{2}\right)} \int_0^{\infty} e^{-t/2} t^{n/2-1} dt \right]$ equals

- (A) 0.5
 (B) 0
 (C) 0.0228
 (D) 0.1587

13. Let X_1 and X_2 be two independent random variables having the same mean θ . Suppose that $E(X_1 - \theta)^2 = 1$ and $E(X_2 - \theta)^2 = 2$. For estimating θ , consider the estimator $T_\alpha(X_1, X_2) = \alpha X_1 + (1 - \alpha) X_2$, $\alpha \in [0, 1]$. The value of $\alpha \in [0, 1]$, for which the variance of $T_\alpha(X_1, X_2)$ is minimum, equals

- (A) $\frac{2}{3}$
 (B) $\frac{1}{2}$
 (C) $\frac{1}{4}$
 (D) $\frac{3}{4}$

14. Let $x_1 = 3, x_2 = 4, x_3 = 3.5, x_4 = 2.5$ be the observed values of a random sample from the probability density function

$$f(x|\theta) = \frac{1}{3} \left[\frac{1}{\theta} e^{-\frac{x}{\theta}} + \frac{1}{\theta^2} e^{-\frac{x}{\theta^2}} + e^{-x} \right], \quad x > 0, \theta > 0$$

Then the method of moments estimate (MME) of θ is

- (A) 3.5
 (B) 4
 (C) 2.5
 (D) 1



15. Let $x_1 = -2$, $x_2 = 1$, $x_3 = 3$, $x_4 = -4$ be the observed values of a random sample from the distribution having probability density function

$$f(x | \theta) = \frac{e^{-x}}{e^{\theta} - e^{-\theta}}, \quad -\theta \leq x \leq \theta, \quad \theta > 0$$

Then the maximum likelihood estimate of θ is

- (A) 3
(B) 0.5
(C) 4
(D) Any value between 1 and 2

16. Let X and Y be independent and identically distributed uniform random variables over the interval $(0, 1)$ and let $S = X + Y$. Find the probability that the quadratic equation

$$9x^2 + 9Sx + 1 = 0 \text{ has no real root.}$$

17.

Find the number of real roots of the polynomial $f(x) = x^5 + x^3 - 2x + 1$.



18. Consider the $n \times n$ matrix

$$P = \begin{bmatrix} \frac{2}{n+1} & \frac{1}{n+1} & \frac{1}{n+1} & \cdots & \frac{1}{n+1} \\ \frac{1}{n+1} & \frac{2}{n+1} & \frac{1}{n+1} & \cdots & \frac{1}{n+1} \\ \frac{1}{n+1} & \frac{1}{n+1} & \frac{2}{n+1} & \cdots & \frac{1}{n+1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{1}{n+1} & \frac{1}{n+1} & \frac{1}{n+1} & \cdots & \frac{2}{n+1} \end{bmatrix}$$

Let I be the $n \times n$ identity matrix and let J be the $n \times n$ matrix whose all entries are 1. Express the matrix P as $aI + bJ$, for suitable constants a and b . It is known that the inverse of P is of the form $cI + dJ$, for some constants c and d . Find the constants c and d in terms of n .

19. Let

$$f(x) = (x + 1) |x^2 - 1|$$

Verify the differentiability of the function $f(\cdot)$ at the points $x = -1$ and $x = 1$.

20. There are four urns labeled U_1, U_2, U_3 and U_4 , each containing 3 blue and 5 red balls. The fifth urn, labeled U_5 , contains 4 blue and 4 red balls. An urn is selected at random from these five urns and a ball is drawn at random from it. Given that the selected ball is red, find the probability that it came from the urn U_5 .

21. Let

$$F(x) = \begin{cases} 0, & \text{if } x < 0 \\ \frac{x^2}{10}, & \text{if } 0 \leq x < 1 \\ \frac{x+2}{8}, & \text{if } 1 \leq x < 2 \\ \frac{c(6x - x^2 - 1)}{2}, & \text{if } 2 \leq x \leq 3 \\ 1, & \text{if } x > 3. \end{cases}$$

Find the value of c for which $F(\cdot)$ is a cumulative distribution function of a random variable X . Also evaluate $P(1 \leq X < 2)$.

22. Let A , B and C be pair-wise independent events such that

$$P(A \cap B) = 0.1 \quad \text{and} \quad P(B \cap C) = 0.2.$$

Show that $P(A^c \cup C) \geq \frac{7}{8}$.






23. Let the random variables X and Y have the joint probability density function

$$f(x, y) = \begin{cases} ce^{-(x+y)}, & y > x > 0 \\ 0, & \text{otherwise} \end{cases}$$

Evaluate c and $E(Y | X = 2)$.

24. Find the area of the region enclosed between the curves $y = (x - 2)^2$ and $y = 4 - x^2$. Or 

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25. Let \bar{X} be the sample mean of a random sample of size 10 from a distribution having the probability density function

$$f(x | \theta) = \frac{1}{\theta} e^{-\frac{x}{\theta}}, \quad x > 0, \theta > 0.$$

Using Chebyshev's inequality, show that $P(0 < \bar{X} \leq 2\theta) \geq 0.9$.



26. Let X be a continuous random variable having the probability density function

$$f(x) = \begin{cases} \frac{2}{25}(x+2), & -2 \leq x \leq 3 \\ 0, & \text{otherwise} \end{cases}$$

Find the cumulative distribution function of $Y = X^2$. Hence derive the expression for probability density function of Y .

27. Let X be a single observation from the probability density function

$$f(x | \theta) = (\theta + 1)x^\theta, \quad 0 < x < 1,$$

where $\theta \in \{1, 2\}$ is unknown. Find the most powerful test of level $\alpha = \frac{13}{49}$ for testing $H_0: \theta = 1$ against $H_1: \theta = 2$. What is the power of the test?





28. Let X_1, X_2, \dots, X_n be a random sample from a population having the probability density function

$$f(x|\theta, \sigma) = \begin{cases} \frac{1}{\sigma} e^{-\frac{(x-\theta)}{\sigma}}, & x > \theta, \\ 0, & \text{otherwise,} \end{cases} \quad -\infty < \theta < \infty, \quad \sigma > 0.$$

Find the method of moments and maximum likelihood estimators of θ and σ .

29. Let

$$f(x) = 3(x-2)^{\frac{2}{3}} - (x-2), \quad 0 \leq x \leq 20.$$

Let x_0 and y_0 be the points of the global minima and the global maxima, respectively, of $f(\cdot)$ in the interval $[0, 20]$. Evaluate $f(x_0) + f(y_0)$.



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