Exo-Atmospheric IR Tracking of Ballistic Objects

M. Srinivasan[1], S. Bhaumik[2], S. Sadhu[3] and T.K.Ghoshal[4]

Abstract – **Infra red seekers are now considered viable alternative to MMW seekers for exo-atmospheric tactical Ballistic Missile (TBM) interceptors. Only gimbal angle and inertial sightline rate signals are available from such seekers. Both of these are corrupted with random noise and residual body motion signals. The inertial or earth reference based bearing angle signal may be obtained from the INS and the seeker gimbal angle. The initial relative co ordinates and velocities may be obtained from the INS and the ground radar. In the penultimate phase of the interception, the interceptor trajectory may be approximated by a straight line making a shallow angle with the near straight line TBM trajectory. The intercepting platform (seeker) moves towards the ballistic object, though the velocities are several magnitudes higher and inertial sightline rate as well as bearing angle are available as measurement. Two cases have been studied. In the first case, only the gimbal angle information is used and three estimation techniques are compared namely Extended Kalman filter (EKF), Unscented Kalman Filter (UKF) and pseudomea surement filter. It is shown that EKF and UKF fail to track in a considerable number of cases but the pseudomeasurement filter has a performance close to the theoretical optimum described by the Cramer Rao bound. In the second case, the raw noisy sight line rate available from the seeker is considered as a second measurement. It is shown that the tracking performance improves substantially.**

Keywords – **Ballistic Target Tracking, EKF, UKF, Pseudo measurement Filter, Monte Carlo simulation.**

I. Introduction

Conceptually, the primary objective of a ballistic object tracking is to estimate the course and velocity of the ballistic object using available sensor measurements. In the last decades, the problem of tracking a ballistic object at the atmospheric region has been an important field of study and has attracted the attention of many researchers [1, 2, 3, 4, 5]. In [2, 3], the problem of tracking a ballistic object, which is to be launched from one point on earth to landing at another point through ballistic path on re-entry using radar measurement has been considered. In [4, 6], we have been studied a vertically falling ballistic object that on reentry has been tracked using ground radar. These kinds of problems have important application for missile defense system and safety of ageing satellites during reentry and in reusable space vehicle.

1,2,3,4 Department of Electrical Engineering, Jadavpur University
Kolkata-700 032, Phone/Fax: +91-33-2414 6723
1msrinivasan77@yahoo.com,_bhaumik_23@yahoo.co.in,
@debesh.wb.nic.in, 4tkghoshal@debesh.wb.nic.in,

This paper adopts a problem of tracking a ballistic object, for example, TBM from interceptor using IR seekers measurements in the penultimate phase of interceptions at exo-atmospheric phase. In this problem the tracking scenario is formulated with the following hypothesis: in exo-atmospheric reentry phase the drag is almost zero and the only acting force is gravity with shallow angle to vertical falling trajectory. With this hypothesis, the interceptor (seeker) trajectory may be approximated by a straight line making a shallow angle with the near straight line TBM (target) trajectory. Hence, this problem is formulated in coordinate frames which is superficially similar to 2-D bearing only tracking simulation scenario [7, 8, 9] and the salient difference is the target and platform moving towards each other with higher magnitude of velocities. Some nonlinear tracking filter techniques including EKF [1, 10], UKF [11-15] and Pseudo-measurement filter [7] are proposed for this problem and the filter performances have been studied using two different initialization methods.

In the first case study (case-I), it is assumed that the gimbal angle (bearing angle) is the only available measurement. The root mean square (RMS) error performance and robustness in the sense of track losses of EKF, UKF and Pseudomeasurement filter are compared based on the large Monte Carlo (MC) simulations and the accuracy of filters is evaluated by Cramer Rao lower bound (CRLB) [2-5] which provides a lower bound on the estimate of variance of unbiased estimators. In this work, the error covariance of three filters would be compared with best achievable mean square error of CRLB.

In the second case study (case-II), the sight line rate (SLR) measurement is also considered along with bearing measurement and the effects in the performance of nonlinear Kalman filters are studied by varying the noise covariance of SLR measurement.

II. Problem Formulation

This section describes the target and platform kinematics for ballistic object and correlated noisy measurement model for two-dimensional target tracking scenario in exo-atmospheric phase. This tracking scenario can be formulated in the geometric x-y coordinate and it is shown in figure 1.

A .Target and platform motion models

As shown in figure 1 we assume that the platform is moving along x-axis with constant velocity. The kinematics of platform motion is given in the following discrete equations

$$x_p(k) = \bar{x}_p(k) + \Delta x_p(k) \tag{1}$$

$$y_p(k) = \bar{y}_p(k) + \Delta y_p(k) \qquad k=0,1,....,N \tag{2}$$

where $\bar{x}_p$ and $\bar{y}_p$ are average platform position co-ordinates and $\Delta x_p(k)$ and $\Delta y_p(k)$ are the mutually independent zero mean Gaussian white noise sequences with variances $r_x = 1\ m^2$ and $r_y = 1\ m^2$, respectively. The mean positions of the platform are: $\bar{x}_p(k)$=10000kt and $\bar{y}_p(k)$ with.T=0.2

As described earlier, the target motion is almost linear with very less process noise (almost zero) at exo-atmospheric phase. Therefore, the target motion is modeled with the following state space relations (discrete-time domain)

$$x(k+1) = Fx(k) + B \tag{3}$$

where $F = \begin{bmatrix} 1 & T \\ 0 & T \end{bmatrix}$ is state transition matrix, $x(k)=[x_1(k)x_2(k)]$ is the position and velocity of the ballistic object at every k^{th} time instant and $B = \begin{bmatrix} 0 \\ -\cos\gamma\, gT \end{bmatrix}$. Here γ is angle between vertical and slant motion of target and it is assumed 60°,

then $B = \begin{bmatrix} 0 \\ -0.5gT \end{bmatrix}$

B. Measurement model

As described earlier, in this problem initially we assume that bearing angle also called the sight line angle (SLA) is the only available measurement with additive noise. Therefore, the measurement equation is given as

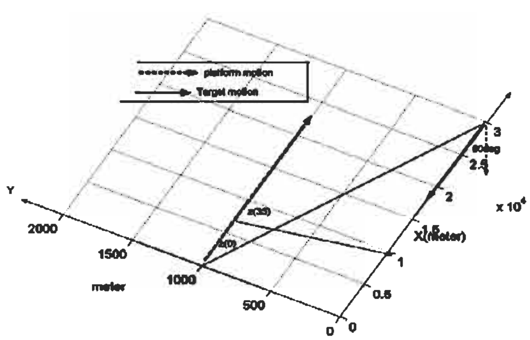

Fig.1. Target kinematics

$$z_m(k) = z(k) + v_s(k) \tag{4}$$

where

$$z(k) = h[x_p(k), y_p(k), x_1(k)] = \tan^{-1}\frac{y_p(k)}{x_1(k) - x_p(k)} \tag{5}$$

is bearing between the horizontal and the line of sight from the sensor (source) to the target and $v_s(k)$ is measurement noise with zero mean and variance $r_s=(0.5^o)^2$ and assume that this noise is independent of the sensor platform perturbations and sampling interval.

For this problem, the random component of platform motion noises can be approximated as additive noise by expanding the measurement equations as

$$\begin{aligned} z_m(k) &= h[x_p(k), y_p(k), x_1(k)] + v_s(k) \\ &\cong h[\bar{x}_p(k), \bar{y}_p(k), x_1(k)] + v(k) \end{aligned} \tag{6}$$

where *v(k)* is equivalent to additive measurement noise (with variance *R(k)*) given approximately by small perturbation theory as

$$v(k) = \frac{[\bar{y}_p \Delta x_p + \{x_1(k) - \bar{x}_p(k)\}\Delta\bar{y}_p]}{[x_1(k) - \bar{x}_p(k)]^2 + \bar{y}_p(k)^2} + v_s(k) \tag{7}$$

R(k) is evaluated by considering that $\Delta x_p(k)$, $\Delta y_p(k)$ and $v(k)$ and mutually independent as

$$R(k) = E[v(k)^2] = \frac{\bar{y}_p(k)^2 r_x + [x_1(k) - \bar{x}_p(k)]^2 r_y}{[\{x_1(k) - \bar{x}_p(k)\}^2 + \bar{y}_p(k)^2]^2} + r_s \tag{8}$$

In this work, we consider the additional measurement SLR (also called bearing rate) that is also available along with bearing angle and it is obtained from the derivative of *z(k)* with additive noise covariance R_{22}[16, 17].

III. Nonlinear Tracking Filters

A. Filter Initialization

In order to track the target with filters it is necessary to provide initial estimates of the state and error covariance matrix. This subsection describes the initialization assumptions, which have been used in this paper for filters.

a) Method-I

Traditionally, the tracking filter is initialized by first two measurements [4,7]. For this problem, method-I for initialization follows the procedure outlined in [7,17]. Accordingly $\hat{x}_2(0) = -2500m/s$ and $p_{22}(0|0) = (500m/s)^2$ for this problem.

b) Method-II

The above initialization method-I is no longer desirable due to large uncertainty in the initial measurement for this problem and therefore, we assume, the filters initial conditions are initialized (arbitrarily) from true values and this is referred to as initialization method-II for this application. In the real world the state component of the target will undergo some perturbations which will be assumed as random noise. Hence, the true value of state components is given as assumed as follows

$$x(0|0)=\left[(30000+w_1)m,(-3000+w_2)m/s\right]^T \quad (13)$$

where the terms w_1 and w_2 denote gaussian noises. It is characterized by zero mean and with initial estimated error covariance given as

$$P(0|0)=\begin{bmatrix} 500^2 m^2 & 0 \\ 0 & 100^2 m^2/s^2 \end{bmatrix} \quad (14)$$

As per [10], the initial state of filter is close to the true values, so we have assumed the initial condition of tracking filter as

$$\hat{x}(0|0)=\left[30000m,-3000m/s\right]^T \quad (15)$$

B. Application of EKF

The EKF is well known and is the conventional recursive minimum mean square error (MMSE) estimator for the application of nonlinear target tracking problems [10]. Since the target dynamic model is linear and measurement is nonlinear, the time update of EKF is same as linear Kalman filter and the measurement update only uses Jacobian matrix of measurement for this problem. See [17] for details.

C. Application of UKF

The UKF is also a recursive MMSE estimator which uses unscented transform (UT) for estimation [11,13]. The UT method transforms one variable to another through nonlinear transformation and computes the statistic of transformed variable. For this application, we have used the scaled version of UKF [14,15]. In this problem as the process model is linear and measurement is nonlinear the UKF algorithm has been simplified suitably. See [17] for more details.

D. Application of Pseudomeasurement Filter

The Pseudomeasurement filter algorithm is based on pseudomeasurement, which is obtained by transforming the original nonlinear measurement to a pseudolinear structure [7]. The pseudomeasurement filter has been implemented as outlined in [17]

IV. CRLB FOR BALLISTIC TARGET TRACKING

It is argued that the CRLB [2, 4, 5, 16, 17] provides a quantitative understanding of signal processing capabilities of the tracking filter performance, without going into the filter design and elaborate MC performance analysis. Actually it is the least mean square error that can be achieved by any optimal unbiased estimators. For the present estimation problem, the lower bound of the RMS error (LBRE) of individual state, given by the square root of the corresponding diagonal elements of Fisher information matrix have been used to compare the performance of the three filters.

V. SIMULATION RESULTS

This section describes the simulation results of proposed nonlinear filters with different initial conditions as described in section 2. The performance comparison is based on large MC simulation. The RMS error performance of proposed three filters with initialization method-I shows there is divergence in EKF for 10000 MC runs and the position and velocity error are given Figure 2 and 3 respectively.

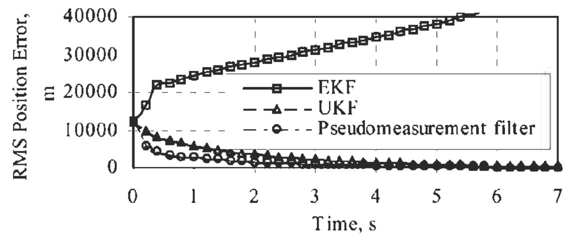

Fig.2. RMS Position Errors

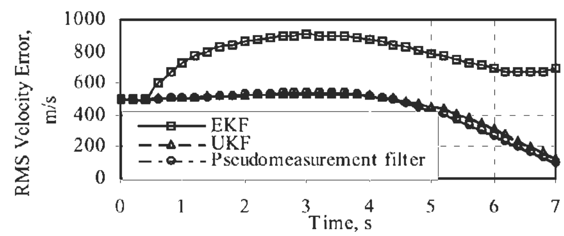

Fig.3. RMS Velocity Errors

As per [7, 8, 9], the track loss was usually associated with large errors in position and velocity. For this application, we have defined the track loss as whenever the absolute position error after 36 cycles failed to settle below to a specified value. Under this condition the track losses are listed in the Table.1 for 10000 MC runs.

It is expected that the initialization of filters based on method-II would be a preferable choice. Using this method two different experiment simulation results are given as below:

A. Case-I: Only SLA measurement

In this experiment, we have studied RMS error performance and track losses of three filters with only bearing angle measurement. We noticed that this problem introduces track loss in EKF and UKF, but there is no track loss in Pseudo measurement for 10000 MC runs and it is tabulated in Table-1.The result for square root of position and velocity estimate error covariance for CRLB and filters are shown in figure 4 and 5 respectively.

B. Case-II: Both SLA and SLR measurements

In this experiment, we incorporate SLR along with bearing angle measurements, and study its effects in EKF and UKF performance with change of variance of R_{22}. The position and velocity RMS error performance are given in figures 6 and 7 for UKF under for different covariance of R_{22} 0.0001,0.001, and0.01$(rad/sec)^2$. Also in this case, there is failure in 1 out of 10000 MC runs in EKF but there are no failure cases in UKF when R_{22}=0.0001 $(rad/sec)^2$. In this case, RMS error performance (excluding failures) of EKF is nearly same as UKF.

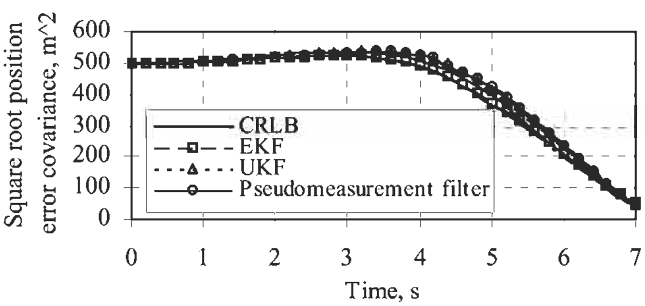

Fig.4. Square Root of Position Error Covariance

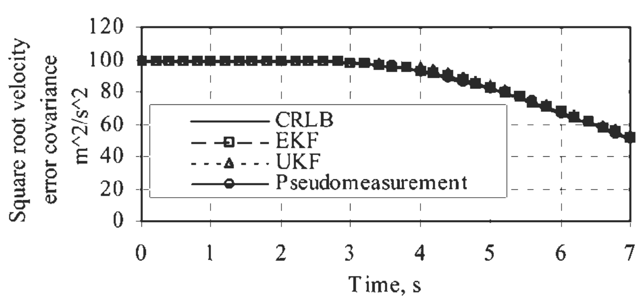

Fig.5. Square Root of Velocity Error Covariance

Table I: Comparison of Track-Loss and CPU Load 1000 MC Runs

Technique Used	TRACK LOSS FOR DIFFERENT INITIAL CONDITIONS		CPU (P4, 1.8GHz) computation al time in sec
	Method -1	Method -2	
1. EKF	95	10	53.86
2. UKF	2	1	215.10
3. Pseudomeasurement Filter	0	0	52.48

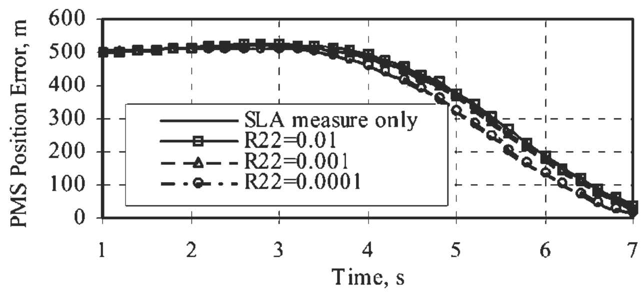

Fig.6. RMS Position Error with effect of SLR measurement

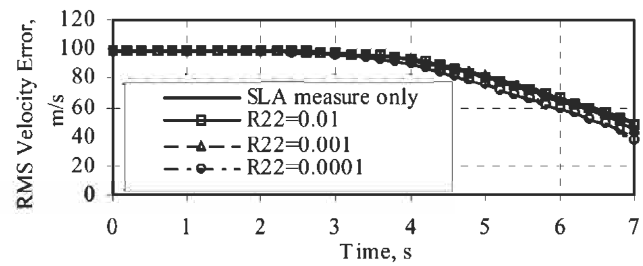

Fig.7. RMS Velocity Error with effect of SLR measurement

VI. Discussions And Conclusion

The problem of tracking a ballistic object in the exo-atmospheric region has been studied by applying nonlinear filtering techniques: EKF, UKF and Pseudomeasurement filter and with different initialization methods.

We compared the RMS error performance as well as the track losses of three filters using only SLA measurement with initialization method-1. The simulation results with method-I indicate that EKF is diverging, but there is no divergence in UKF and Pseudomeasurement filter. According to the track loss analysis, there are more track losses in EKF than UKF, but there is no track loss in Pseudomeasurement filter. From these we conclude that Pseudomeasurement is better than EKF and close to UKF in performance.

On the other hand, in initialization with method-II, the performance of the same filters has been obtained by MC simulation. With this initial condition, we investigated the filter performance by using two kinds of measurements: only bearing angle and both bearing angle and bearing rate. Using only bearing measurement, the CRLB has been derived and the covariance of these three filters has been investigated. The position and velocity of square root estimate error covariance performance of these three filters were compared. The error covariance of Psuedomeasurement filter is close to the CRLB and it indicates that the Psuedomeasurement filter gives good performance.

We have also investigated EKF and UKF performance with SLR measurement along with SLA. We found that the RMS error performance of UKF is relatively better than the EKF.

From the point of view of robustness, computational time and conversion complexity, the authors are of the opinion that the Pseudomeasurement filter is best suited for this application.

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