Tracking of Ballistic Target on Re-entry Using Ensemble Kalman Filter

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***Abstract*—In this work, ground radar based ballistic target tracking problem in endo-atmospheric re-entry phase with unknown ballistic coefficient has been solved using ensemble Kalman filter (EnKF). EnKF, a powerful tool in nonlinear estimation, is being extensively used by meteorologist but almost unknown to target tracking community. Performance improvement, and computational burden of EnKF with increasing ensemble size have been studied. Performance of EnKF has been compared with most popular extended Kalman Filter (EKF) in terms of biasness, estimation accuracy, and computational efficiency. The simulation results reveal that the estimation accuracy of EnKF with sufficient ensemble size is much better than EKF.**

I. Introduction

The tracking problem of a ballistic target on re-entry has attracted a large number of researchers during last four decades. Probably the first attempt has been made by Athans and others [1] to estimate states of a freely falling particle using nonlinear filter. The problem had received immediate attention from research community [2], [3], [4], [5] and it has been solved using extended Kalman filter (EKF) and its variants. A detailed survey on gradual improvement of re-entry target tracking techniques over last four decades has been presented in [6].

With the advancement of nonlinear estimation technique and computing facility, the problem has gained renewed interest and momentum. Specially during the first decade of twenty first century, many attempts [7], [8], [9], [10], [11] have been made to find out accurate and computationally effective tracking filter to track a ballistic target. The researchers used available advanced nonlinear estimators such as unscented Kalman filter [9], particle filter [7], [9], quadrature based filters [12], *etc* and compared the estimation accuracy among them and with the Cramer Rao lower bound [9].

Tracking of a ballistic target in endo-atmospheric region is difficult because some physical parameters of ballistic object, such as ballistic coefficient, drag, lift *etc.* are not known with sufficient accuracy [5] and local atmospheric conditions are also not precisely predictable. Moreover, the model of ballistic target on re-entry becomes nonlinear [13] in nature ruling out the possibility of existing any optimal estimate of position and velocity. The challenging nature of the problem, discussed above, forces the researchers to develop more accurate and computationally efficient nonlinear tracking filter.

In a development, Evensen [14], [15] proposed a nonlinear estimator named as ensemble Kalman filter (EnKF) in the context of weather forecasting [16]. Although the estimator, which is very popular to meteorologist, is able to produce accurate, and computationally effective estimation, even for a system with very high dimension, it is unknown to target tracking community (except the work of Cui [17]). EnKF is a kind of numerical Monte Carlo (MC) simulation method, used to solve intractable integrals. In EnKF, the ensemble of adequate size is formed with the states sampled from the posterior probability density function and measurements. The ensemble is propagated with the help of state and measurement equations and updated with Kalman filter scheme.

In this paper, an attempt has been made to retrieve the EnKF from meteorologist domain and reformulate with the signal processing vocabulary. The algorithm is applied to a typical ballistic target tracking on re-entry problem. The results obtained from EnKF have been compared with traditional extended Kalman filter in terms of estimation accuracy, biasness in estimation, and computational efficiency. The improvement of estimation accuracy with increasing size of ensemble has also been reported.

II. Problem Formulation

The entire trajectory of a ballistic target, from launch to impact, is commonly divided into boost, coast and re-entry phases [13]. Here, we consider tracking of a ballistic target using ground radar measurement in re-entry phase. When the ballistic target re-enters in the atmosphere, the speed of the target is very high and time to hit the ground is very small allowing small time for estimator to converge.

A. Target Model

In this paper, the target is assumed to be falling vertically downward as shown in Fig 1. So the motion of the target becomes single dimensional in nature. We consider the case where ballistic coefficient (β), which is a function of mass, and shape, is unknown. The main objective is to estimate the position, velocity, and ballistic coefficient of the target with the help of ground radar measurement. We adopt the state space formulation of the problem as described in [9], [12]. We evaluate the motion of the target with the assumption that

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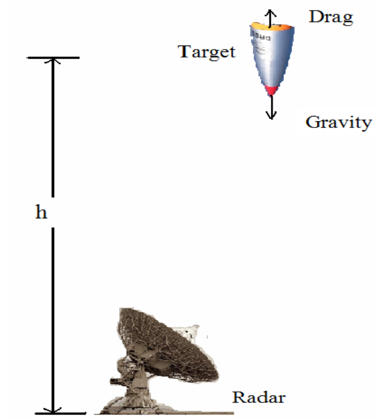

Fig. 1. Ballistic target tracking scenario using ground radar

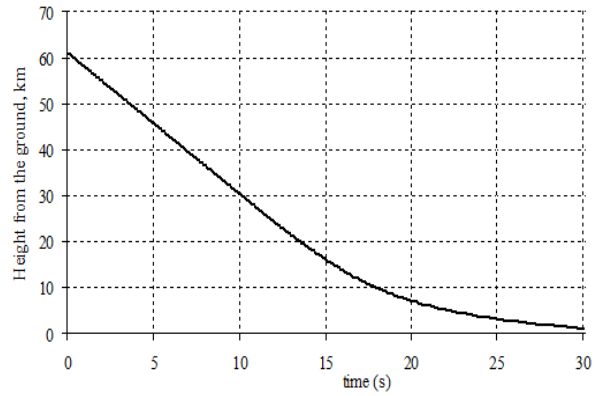

Fig. 2. Typical target trajectory-altitude vs time

only drag and gravity are the two forces acting on it. We neglect lift and all other forces. The gravity is also considered to be fixed with respect to altitude.

Under the above described assumptions, the kinematics of the target is governed by the following continuous time domain equations.

$$\dot{h} = -v \tag{1}$$

$$\dot{v} = -\frac{\rho(h)gv^2}{2\beta} + g \tag{2}$$

$$\dot{\beta} = 0 \tag{3}$$

Where, h is altitude (position) in meter, v is velocity in m/s, $\rho(h)$ is air density, g is acceleration due to gravity ($9.81m/s^2$) and β is the ballistic coefficient. Air density is exponential function of altitude and is given by

$$\rho(h) = \gamma e^{-\eta h} \tag{4}$$

Where, $\gamma = 1.754$, and $\eta = 1.49 \times 10^{-4}$. Now, to implement the state estimator we discretise the target dynamics. The discretise state space model is given by

$$x_{k+1} = f(x_k) + w_k \tag{5}$$

Where $f(x_k)$ is a nonlinear function used to describe the state evolution and can be expressed as

$$f(x_k) = \phi x_k - G[D(x_k) - g] \tag{6}$$

where x_k, ϕ and G are given in terms of sampling time τ as $x_k = \begin{bmatrix} h & v & \beta \end{bmatrix}^T, \phi = \begin{bmatrix} 1 & -\tau & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, G = \begin{bmatrix} 0 & \tau & 0 \end{bmatrix}^T$. The drag is expressed as

$$D(h_k, v_k, \beta_k) = \frac{g.\rho(h_k).v_k^2}{2\beta_k} \tag{7}$$

It is to be noted that the nonlinearity in target dynamics arises due to drag only. w_k is process noise arises due to imperfection in kinematics model. The process noise is assumed as Gaussian, and characterized by zero mean, Q_k covariance

$$Q = \begin{bmatrix} q_1\frac{\tau^3}{3} & q_1\frac{\tau^2}{2} & 0 \\ q_1\frac{\tau^2}{2} & q_1\tau & 0 \\ 0 & 0 & q_2\tau \end{bmatrix} \tag{8}$$

Where q_1 in (m^2/s^3) and q_2 in $(kg^2/m^{-2}s^{-5})$ are the tuning parameters to be selected by designers to model the process noise in target dynamics.

B. Measurement Model

The radar located on the ground is assumed to measure the altitude of target at any instance of time. The measurement equation is assumed to be linear and corrupted with white Gaussian noise.

$$y_k = Hx_k + \eta_k \tag{9}$$

Where $H = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$, and η_k is zero mean white Gaussian measurement noise with variance R_k.

C. Typical target trajectory

A typical target's altitude, velocity and acceleration are plotted in Fig. 2 to Fig. 4 respectively. No process noise is assumed. So $q_1 = 0$, and $q_2 = 0$. Initial heights and initial velocity are taken as 60960m and 3048m/s respectively. Ballistic coefficient and sampling time are taken as $19161kg/ms^2$ and $\tau = 0.1$ second respectively.

III. Ensemble Kalman Filter

Ensemble Kalman filter (EnKF) was proposed by Evensen [14] in 1994, for large dimensional severely nonlinear problem in the context of weather forecasting. It is a kind of numerical estimation technique, may be thought of as Monte Carlo implementation of the Kalman filter for nonlinear estimation problems. Since the birth of EnKF, it has extensively been used to estimate dynamic variables in large scale atmospheric, oceanographic model [15] and weather forecasting. Later it

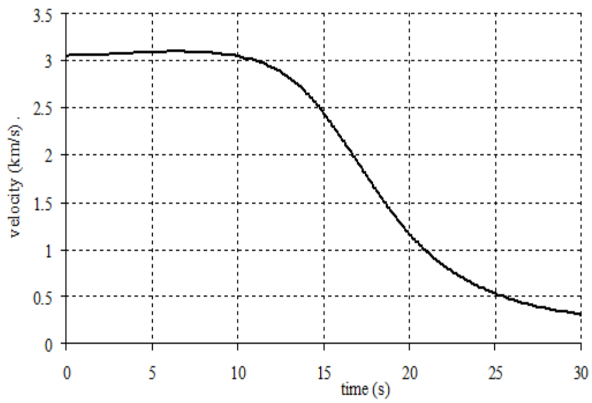

Fig. 3. Typical target trajectory- velocity vs time

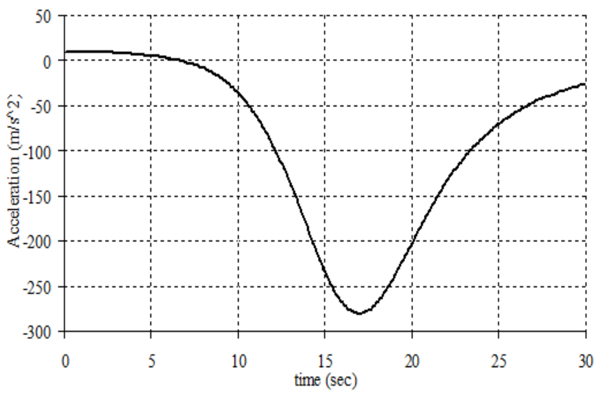

Fig. 4. Typical target trajectory - acceleration vs time

has been adopted to estimate reservoir production index [18] in petroleum industry. However the application of EnKF in target tracking problem is rare (except the work of Cui *et. al.* [17]).

The implementation of EnKF is based on the following steps:

(i) Generation of Ensemble points of augmented system (states of the system are augmented by incorporating measurements).

(ii) Propagation of ensemble points through process and measurement equations. Evensen called the step as *forecast* which is commonly known as *predictor* step.

(iii) Updates of ensemble points using Kalman filter scheme with the help of measured data and measurement noise statistics. The step is commonly known as *analysis* by weather scientists, whereas system engineers is habituated to call it as *measurement update*.

A. Comparison with Particle filter

The EnKF is a Monte Carlo simulation method used to estimate the states of a system. Another numerical technique for state estimation, named as particle filter (PF) [19], is available in literature. The PF method is very popular to designer. EnKF which is not well known to system engineers and target tracking community possess several computational advantages compared to particle filter.

(i) Computationally, particle filter is not suitable for very high dimensional problem. With the increase in dimension, the number of particles needed to obtain reasonable accurate estimation increases very rapidly. On the contrary, ensemble size of 50-100 is sufficient for any dimensional problem [20].

(ii) The prior ensemble is approximated with Gaussian distribution which allows Kalman filter scheme to generate posterior ensemble points. The particle filter [19] approximates the probability density function with particles in state space and weight associated with them. It performs the update step with the help of likelihood function obtained from measurement. No concept of weight is associated with EnKF.

(iii) Unlike PF, in EnKF computationally expensive resampling step is not necessary. In EnKF, the posterior ensemble is directly propagated to next step.

B. EnKF Algorithm

In this subsection we summarize the steps associated with EnKF algorithm. Let us assume nonlinear target dynamics, and measurement equation are described as,

$$x_{k+1} = f(x_k) + w_k$$

$$y_k = h(x_k) + \eta_k$$

Where $x_k \in R^n$ and $y_k \in R^p$ denote the states and the measurements of the system respectively at any instant k, where $k = \{0, 1, 2, 3, ...\}$. $f(.)$ and $h(.)$ are known nonlinear function of x and k. $w_k \in R^n$ and $\eta_k \in R^p$ are process and measurement noise respectively. η_k is zero mean white Gaussian with covariances R_k.

Step (i) Filter initialization

- State vector is augmented with measurement to form ensemble [20]. Initially an ensemble of size N is defined as

$$\chi_0 = \begin{bmatrix} X_0 \\ Y_0 \end{bmatrix} = \begin{bmatrix} x_0^1 & x_0^2 & x_0^3 & ... & x_0^N \\ y_0^1 & y_0^2 & y_0^3 & ... & y_0^N \end{bmatrix}$$

where $\chi_0 \in R^{(n+p)\times N}$, $X_0 \in R^{n\times N}$, and $Y_0 \in R^{p\times N}$. x_0^j are drawn from the initial distribution and y_0^j are obtained from $h(x_0^j)$.

Step (ii) Predictor step

- Propagate ensemble members

$$X_{k|k-1} = f(x^j_{k|k}) + w_{k-1}$$

$$Y_{k|k-1} = h(x^j_{k|k})$$

- Evaluate the ensemble error covariance

$$P_{k|k-1} = L_{k|k-1} L^T_{k|k-1}$$

where

$$L_{k|k-1} = \frac{1}{\sqrt{N-1}}[(\chi^1_{k|k-1} - \hat{\chi}_{k|k-1}) \quad (\chi^2_{k|k-1} - \hat{\chi}_{k|k-1}) \quad \cdots \quad (\chi^N_{k|k-1} - \hat{\chi}_{k|k-1})]$$

and

$$\hat{\chi}_{k|k-1} = \frac{1}{N}\sum_{j=1}^{N} \chi^j_{k|k-1}$$

Step (iii) Corrector step or measurement update

- Calculate Kalman gain matrix

$$K_k = P_{k|k-1}H^T(HP_{k|k-1}H^T + R_k)^{-1}$$

where H matrix is given as $H = [0_{1\times n} \quad I_{1\times p}]$.

- Update ensemble points

$$\chi^j_{k|k} = \chi^j_{k|k-1} + K_k(y^j_{k,o} - y^j_{k|k-1})$$

where, $y^j_{k,o}$ are obtained from the distribution of measurement noise, $y^j_{k,o} \sim \aleph(y_k, R_k)$.

Step (iv) EnKF estimate

- Calculate mean

$$\hat{\chi}_{k|k} = \frac{1}{N}\sum_{j=1}^{N} \chi^j_{k|k}$$

- Estimate states $\hat{X}_{k|k} = G\hat{\chi}_{k|k}$. Where G is a matrix given by $G = [I_{1\times n} \quad 0_{1\times p}]$

Remarks

- For a nonlinear process and linear measurement system, the ensemble can be formed with the states only. Correspondingly update equation of ensemble needs little modification.
- In should be noted that ensemble variance is calculated by dividing with N-1 instead of N, because the population mean is unknown. The correction is known as *Bessel's correction*. It reduces the bias in the estimation of the population variance [21].
- In EnKF algorithm, there is no need of linearization. It may be considered as an added advantage.

IV. Simulation Results

A. Initialization

The truth trajectory of the ballistic target is simulated with the initial height and velocity as $h = 60960m$, and $v = 3048m/s$ respectively. In this paper, we follow the initialization scheme as described in [12]. The ballistic coefficient (β) is initialized from beta distribution with both the parameters 1.1, *i.e.* $\beta \sim Be(1.1, 1.1)$, with upper and lower level boundaries are at $10,000kg/ms^2$ and $63,000kg/ms^2$ respectively. Ballistic coefficient is initialized randomly as there is no clue available about the shape and size of the ballistic target to be tracked.

The filter or estimator is initialized with the random number generated from Gaussian distribution with mean and covariance as $\hat{x}_{0|0} = [60960 \quad 3048 \quad mean(\beta_0)]^T$, and

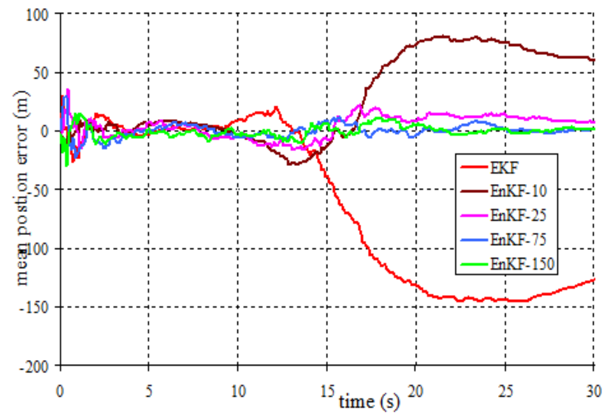

Fig. 5. Mean error of position out of 100 MC runs obtained from EKF and EnKF with different ensemble size

$P_{0|0} = \begin{bmatrix} R_k & \frac{R_k}{\tau} & 0 \\ \frac{R_k}{\tau} & \frac{R_k}{\tau^2} & 0 \\ 0 & 0 & \sigma^2_\beta \end{bmatrix}$. β_0 and σ_β are the mean and standard deviation of the random number generated from beta distribution, described above.

B. Results

The above described ballistic target tracking problem has been solved using extended Kalman filter (EKF) and EnKF. The sampling time τ is taken as 0.1 and simulation is performed for $30sec$ duration of time. The process noise for both truth and estimator is taken as described in [12] with $q_1 = q_2 = 5$. The measurement noise covariance (R_k) is taken as $R_k = 200^2$. The simulation is carried out with different sizes of ensemble and results are compared for 100 Monte Carlo runs. The mean of estimation error of position, velocity, and ballistic coefficient (β) obtained from EKF and EnKF (with different ensemble sizes) are shown in Fig. 5 to Fig. 7 respectively. We observe from the figures that the EKF and EnKF with small ensemble size exhibits comparatively large bias in estimation of hight, velocity, and ballistic coefficient of target. The EnKF with ensemble size more than 50 shows unbiased altitude and velocity estimation. For ballistic coefficient estimation, the EnKF with same ensemble size shows unbiasedness after 20 second of simulation time.

The standard deviation of altitude, velocity, and ballistic coefficient obtained from EKF and EnKF (with different ensemble points) out of 100 MC runs are plotted in Fig. 8 to Fig. 10 respectively. From Fig. 8, it has been observed that the standard deviation of position error obtained from EKF diverges which indicates, with EKF, tracking of ballistic target is quite difficult to ensure. From the figures, we also observe the performance of EKF is much worse compared to EnKF with sufficient ensemble size. For EnKF, with the increase of ensemble size, the accuracy in estimation increases. However no appreciable improvement in estimation is observed beyond ensemble size of 100.

The simulation results indicate that the performance of

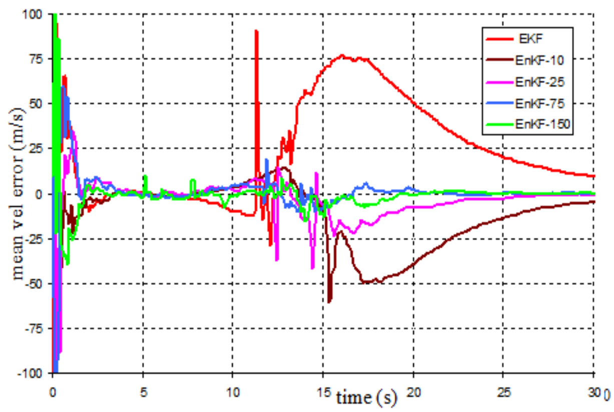

Fig. 6. Mean error of velocity out of 100 MC runs obtained from EKF and EnKF with different ensemble size

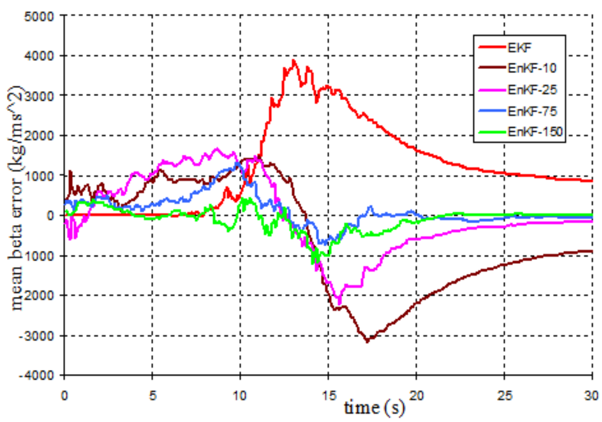

Fig. 7. Mean error of β out of 100 MC runs obtained from EKF and EnKF with different ensemble size

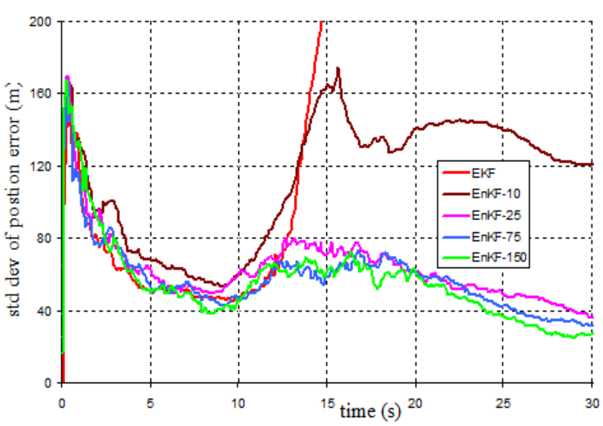

Fig. 8. Std. of position estimation error obtained from 100 MC runs

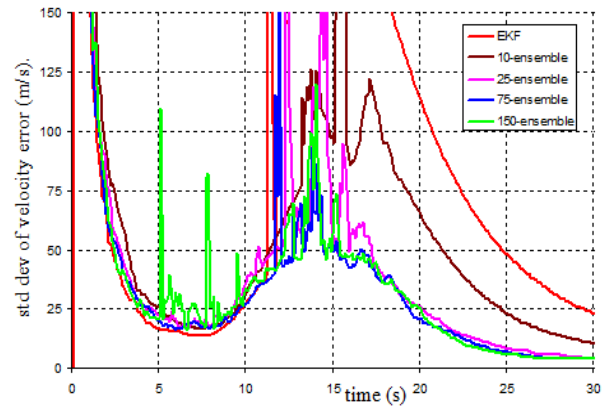

Fig. 9. Std. of velocity estimation error obtained from 100 MC runs

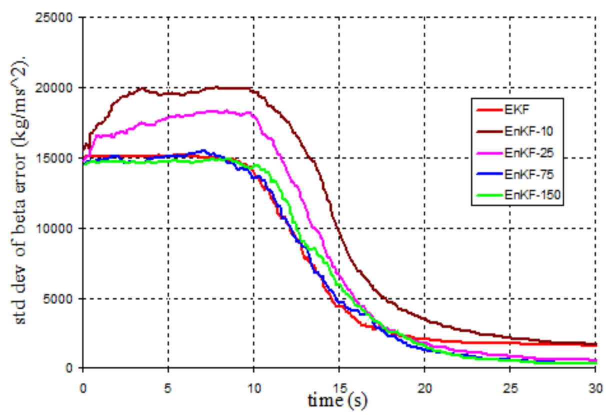

Fig. 10. Std. of ballistic coefficient estimation error obtained from 100 MC runs

EnKF with ensemble size 75 (EnKF-75) is far better than that of obtained from EKF. However we acknowledge the fact that the computational time is much more for EnKF than EKF. In table 1, we compare the computational load of EnKF with different ensemble sizes with that of EKF. The data point in table shows that the EnKF of 75 ensemble size, beyond which appreciable improvement of estimation accuracy is not achieved, is nearly 26 times computationally slower than EKF.

V. Discussions and Conclusion

In this paper, we solve a ballistic target tracking problem using ensemble Kalman filter algorithm which was initially developed by meteorologist to solve nonlinear estimation problem for very high dimensional system and unknown to target tracking community. The performance of EnKF has been compared with traditional EKF in terms of estimation accuracy and computational efficiency. Simulation results reveal that the estimation accuracy of EnKF with sufficient number of ensemble points is far better than EKF. The performance comparison of EnKF with particle and quadrature based filters remains under the scope of future work. Finally, we recommend to use

TABLE I
NORMALIZED COMPUTATIONAL COST OF EKF AND ENKF

Filter	Normalized comp. time
EKF	1
EnKF-10	4.46
EnKF-25	9.47
EnKF-75	26.17
EnKF-150	50.99

EnKF for endo-atmospheric ballistic target tracking on re-entry problem compared to EKF, provided increase in computational time is acceptable.

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