Evolving Homing Guidance Configuration with Cramer Rao Bound

S.Sadhu*, S.Bhaumik and T.K.Ghoshal
Dept of Electrical Engg, Jadavpur University, Kollkata 700032, India
*E-mail: smita@debesh.wb.nic.in

Abstract— ***This paper advocates the use of Cramer Rao Bound (CRB) as a tool to aid decisions on guidance system configuration for homing missiles. It is argued that the CRB provides a quantitative understanding of the influence of model parameters and instrumentation/ signal processing capabilities of the tracking filter performance, without going into the specifics of filter design and elaborate Monte Carlo performance analysis. The concepts have been demonstrated by a bearing only target-tracking (BOT) problem. The CRB performance for position and velocity error has been studied with respect to (i) variation of the sampling time (ii) simplistic change in the measurement noise variance (iii) the effect of introducing additional measurement with different noise variances (iv) the effect of eclipsing on measurement. The example demonstrates that the CRB analysis provides a good handle for tracking system design trade off. The situations where CRB results may mislead are also indicated***

Index Terms— **Cramer Rao Bound, Bearing only Tracking, Homing Guidance.**

I. INTRODUCTION

THERE has been a renewed interest in homing guidance and on board seekers for intercepting tactical ballistic missiles (TBM)[5]. Majority of current interceptors employ MMW seekers with high PRF with consequent advantages and problems. One of such problems is a kind of periodicity in the Signal-to-noise ratio (SNR) due to a phenomenon called eclipsing [2,15].

The seeker usually provides the line of sight (LOS) angle and the LOS rate with respect to the interceptor body frame or inertial frame for guidance [1,13]. The LOS angle is obtained from the body altitude, gimbal angle and the bore-sight error [1,13,14]. In traditional seekers [14] no explicit state estimation is usually performed and the LOS rate is approximated from the servo demand signal and needs further filtering to ensure high probability of target interception.

For TBM interceptor, guidance configuration selection involves, selection/specification of seekers, navigation systems, midcourse (command guidance), homing guidance law, guidance signal filtering and signal fusion. Guidance signal filtering filter options may become as important as the guidance law design. In contrast to yesteryears when Extended Kalman Filter was probably the only choice available for non-linear tracking problems, a variety of filters [5] like Unscented Kalman Filters and Particle filters are available today.

The configuration selection has to be carried out at an early phase of the design life cycle of interceptor, when even the airframe and propulsion system of the interceptor may still be evolving and subsystem data may be only partially available. The design/selection toolset for this phase should have been entirely different from the detailed design and validation tools. In reality, a team either depends on the intuitive feel of an experienced designer or goes for a detailed analysis. For example, the effects of faster sampling, lower measurement noise etc. on filtering performance are known qualitatively, for specific non-linear problems, it is difficult to estimate even "ball park" quantitative results without extensive Monte Carlo simulation. Proper Monte Carlo analysis with 6DOF simulation mode, need far more design details than available in the configuration selection phase.

We therefore advocate taking a middle road between the "gut feelings" of a veteran and a detailed MC study. Use of Cramer Rao bounds (CRB) fits our requirement admirably.

Posterior Cramer Rao Bounds for state estimation of non-linear multivariable dynamic system has gained fairly wide patronage among underwater weapon system designers, mainly due to the work of Kerr [7] and Tichavsky [6]. We would call such techniques by the generic name CRB. Of late, aerospace system designers [5,8,3] have also exhibited interest in CRB.

The CRB gives an indication of the achievable lower bound of the error covariance and acts as a benchmark to demonstrate efficiency of a specific filter [5], especially so, as the filters for non-linear problems are suboptimal. CRB also permits design trade off analysis with performance improvement versus cost of improved instrumentation and signal processing equipment [10].

We extend the current use of CRB analysis to that of a *tool* to assist guidance *configuration design* for homing interceptors.

Using the error covariance information from the CRB studies and the adjoint method [13] permit a rapid evaluation of a candidate estimation/fusion system and guidance algorithm pair. Implicit in this is the assumption is the "pessimism factor" which indicates how worse our real life estimation algorithm be, compared to the best possible theoretically estimator.

Due to the paucity of TBM interception problems in the public domain, we demonstrate the power of CRB analysis with the help of a well known bearing only tracking (BOT) problem [9, 11, 12].

Admittedly, a poor substitute for a TBM engagement problem, the BOT problem selected has similarities in the kinematic situation as the target and the interceptor move in nearly parallel path, involving large and rapid change of sight line angle. Dissimilarities are also glaring. In the TBM engagement the interceptor and target move towards each other, rather the interceptor chasing the target as in the BOT example. The distances and the velocities are about three orders of magnitude higher for the case of the TBM

0-7803-8689-2/04/$20.00 ©2004 IEEE

engagement.

II. BOT Problem Formulation

A. State Model

The earliest version of this particular BOT problem occurs in Bar-Shalom [9] and is later re-formulated by Lin et al [12]. An overview of the problem is given in [10].

The tracking is done using sight line angle measurement from the interceptor. The target is assumed to move in a straight line along the X-axis (fig-1), with a constant average velocity (perturbed by the process noises). The platform, in order to track the target, attempts to move in a course parallel to that of the target with a near constant velocity in the same vertical plane. The average motions of the platform are known and imperfections are modelled by noises in X and Y directions.

The bearing measurement is processed to estimate the velocity and position of the target in an earth fixed coordinate system. The observer model is non-linear and both process and measurement noises are assumed to be Gaussian. Note that the platform motion noise would appear as a measurement noise in an earth fixed frame.

The target is moving in X direction with the following discrete state space relations,

$$\mathbf{x}_{k+1} = \mathbf{F}_k \mathbf{x}_k + \mathbf{G}_k w_k \quad \ldots \quad \ldots (1)$$

with $\mathbf{x}_k = \begin{bmatrix} x_{1,k} \\ x_{2,k} \end{bmatrix}$, $\mathbf{F}_k = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix}$ and $\mathbf{G}_k = \begin{bmatrix} T^2/2 \\ T \end{bmatrix}$

where $x_{1,k}$ is the position along X-axis in meters , $x_{2,k}$ is the velocity in m/sec, w_k is independent zero mean Gaussian white acceleration noise sequence with variance *q*. The sampling time, *T*, has a nominal value of 1sec. The

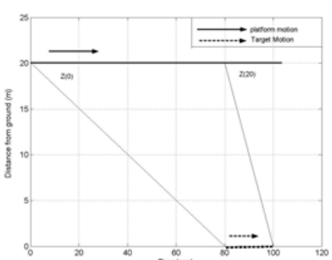

Fig.1 Tracking Platform Kinematics

(unknown) true initial condition is $\mathbf{x}_0 = [80,1]'$. The target motion noise covariance matrix may be computed, with $q = 0.01\, m^2/s^4$ as $\mathbf{Q}_k = \mathbf{G}_k \mathbf{G}_k^T q = \begin{bmatrix} T^4/4 & T^3/2 \\ T^3/2 & T^2 \end{bmatrix} q$

The tracking platform motion may be described as:

$$x_{p,k} = \bar{x}_{p,k} + \Delta x_{p,k} \qquad k=0,1,\ldots,n_{step} \quad \ldots \quad (2)$$

$$y_{p,k} = \bar{y}_{p,k} + \Delta y_{p,k} \qquad k=0,1,\ldots,n_{step} \quad \ldots \quad (3)$$

Where $\bar{x}_{p,k}$ and $\bar{y}_{p,k}$ are the (known) average platform position co-ordinates and $\Delta x_{p,k}$ and $\Delta y_{p,k}$ are the mutually independent zero mean Gaussian white noise sequences with variances $r_x = 1 \times T^2\ m^2$ and $r_y = 1 \times T^2\ m^2$, respectively. The mean positions of the platform are: $\bar{x}_{p,k} = 4kT$ and $\bar{y}_{p,k} = 20$.

B. Measurement Models

In *measurement model-I*, the measurement equation (in bearing coordinate) is given as $z_{m,k} = \bar{z}_k + v_k$ (4)

Where $\bar{z}_k = \tan^{-1} \dfrac{\bar{y}_{p,k}}{\bar{x}_{1,k} - \bar{x}_{p,k}}$ is the average LOS and v_k is the equivalent additive measurement noise (with variance $\mathbf{R}_k$).

In measurement model-1, the measurement noise covariance is given approximately by small perturbation theory as

$$\mathbf{R}_k = \mathrm{E}\left[v_k^2\right] = \frac{\bar{y}_{p,k}^2 r_x + [x_{1,k} - \bar{x}_{p,k}]^2 r_y}{\{[x_{1,k} - \bar{x}_{p,k}]^2 + \bar{y}_{p,k}^2\}^2} + r_s \quad \ldots \quad (5)$$

(with the nominal value of $r_s = \left(3^0\right)^2$),

In *measurement model-II*, the LOS rate measurement is also available along with the LOS. Though in reality, the LOS and the LOS rate measurement noises may be strongly correlated, we assume a diagonal **R** matrix with the element R_{22} being made a variable for a systematic study.

In *measurement model-III*, a high PRF RF seeker was assumed. Range and range rate signals are neglected but the effect of eclipsing [15] was taken as shown qualitatively in fig 6. The consequent $\mathbf{R}_k$ is shown in fig 7.

C. Filter Initialisation

As in [12], The initial position estimate was assumed to be obtained from the first LOS measurement and consequently the covariance is

$$P_{11,0} = r_x + \frac{r_y}{\tan^2 z} + \frac{\bar{y}_p^2}{\sin^4 z} r_s \quad \ldots \quad \ldots \quad (6)$$

Also $\hat{x}_{2,0} = 0$, $P_{22,0} = 1$,. $P_{12,0} = P_{21,0} = 0$

III. Cramer Rao Bound

The posterior CRB may be utilised as follows [6]. Given a state estimation problem, the lower bound of the RMS error (*LBRE*) of individual state are given by the square root of the corresponding diagonal elements of Fisher information matrix $\mathbf{J}_k = \mathbf{P}^{-1}_{mv,k}$ (where $\mathbf{P}_{mv}$ is the minimum variance covariance matrix, often referred as the CRB matrix) :

$$(LBRE)\,\{x_j(k)\} = \sqrt{\mathbf{J}_{jj,k}^{-1}} \quad \ldots \quad (7)$$

IV. RESULTS OF CRB STUDIES

A. Influence of the sampling time on CRB

It is expected that the LBRE would decrease with faster sampling and this general trend is confirmed in Fig 2 where the LBRE of velocity is plotted for different sampling times. However, the effect is far from proportional. For example, the time required for the velocity LBRE to settle to 0.6 m/s are respectively 14s (for T=1s), 11.5s (for T=0.5s), 9.2s (for T=0.2s). The effect of sampling time on position LBRE (not shown) however is nearly 'proportional'. Measurement Model-I has been used for this study. Similar results were obtained with EKF[4] and UKF [11]. The trend shown in fig2 and the fact that the measurement error covariance tends to increase when sampling time is reduced, indicate that reduction of sampling time, beyond a certain point may not be justified.

B. Influence of measurement error covariance

The effect of substantial changes in measurement error covariance (with measurement Model-I) is evident from the expression of the CRB matrix. With decrease in $\mathbf{R}_k$, $\mathbf{J}_{k+1}$ increases, thus decreasing the LBRE of position and velocity. Figs 3 and 4 show that when the measurement error covariance is 1/5th of the nominal value, the terminal position error reduces to 0.48 m compared to 1.01 m in the nominal case. Terminal velocity error reduces to 0.2 m/s compared to 0.29 m/s in the nominal case. Similar trends were obtained with PF [4], EKF [4] and UKF [11].

C. Adding another measurement

Sightline Rate signal is usually available from the seeker. It would be of interest to know whether use of such signal would improve the estimation accuracy. Measurement Model-II and a sampling time of 0.1 s have been used for this study. Three cases are considered for SLR noise covariance (R_{22}) of 0.01, 0.001 and 0.0001 rad/s^2. The last figure is equivalent to the one sigma (1σ) error of 10 mil/s and considered fair for this class of seeker. While introducing this additional measurement has hardly any impact on the LBRE of position, the same for velocity shown in fig5 indicates substantial improvement over the single measurement case (with only sight line angle measurement) when R_{22} is small (<0.01). For larger value of R_{22} the results are the same as that for single measurement.

D. Eclipsing Effect

The eclipsing effect [15] is studied with the LOS-only measurement scheme (Measurement Model III). The SNR for a high PRF, MMW seeker is shown qualitatively in fig 6. Note that as the range decreases, the SNR improves as shown by the envelope of the curve. The eclipsing is characterised (approximately) by an absolute sine wave modulated by the SNR envelope. The corresponding error covariance matrix in logarithmic scale is shown in fig 7. The effect of eclipsing, as contrasted to the same for the SNR envelope on the LBRE of the position and velocity is shown in fig 8 and 9. It may be noted that in this example, the effect of eclipsing is not as severe as one would apprehend.

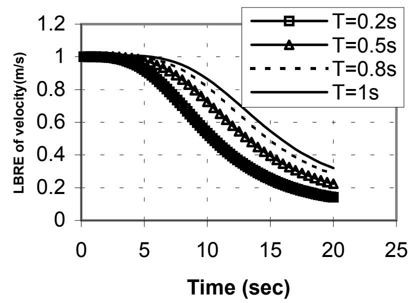

Fig.2. Influence of the sampling time on LBRE of velocity

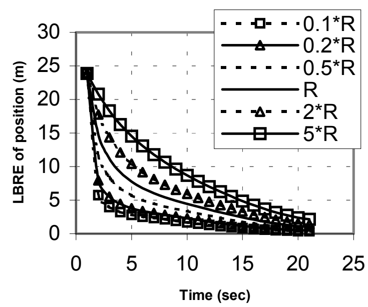

Fig.3. Influence of measurement error covariance on LBRE of position

E. Discussions

The CRB computation is recursively done as in [3,6,10]. The velocity LBRE is emphasised as this has the highest influence on the guidance signal, namely the LOS rate.

A caveat regarding limitation of CRB results is in order. The CRB results should not be used as a guide in situations where the initial error distribution and process noise covariance for the truth and the estimator are not identical, which is sometimes done to "improve" tracking performance.

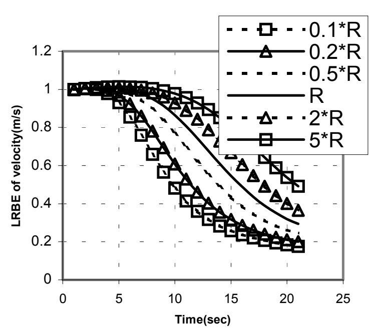

Fig.4. Influence of measurement error covariance on LBRE of velocity

V. CONCLUSION

The role of CRB for a quantitative understanding of the influence of decision parameters on the tracking filter performance has been demonstrated. The potential of using Cramer Rao Bound (CRB) as a tool to aid guidance system configuration design has thereby been established. This has been achieved without going into the specifics of filter design and elaborate Monte Carlo performance analysis for evaluating such filters.

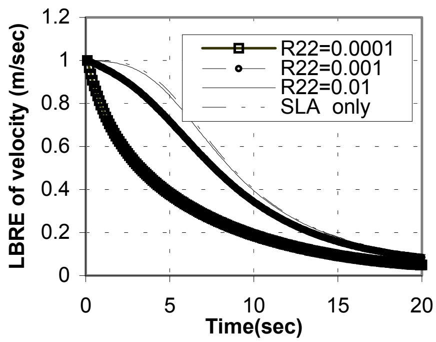

Fig-5. Effect of introducing Sight line rate measurement (velocity)

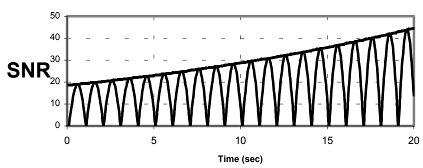

Fig. 6. Signal to noise ratio (with and without eclipsing)

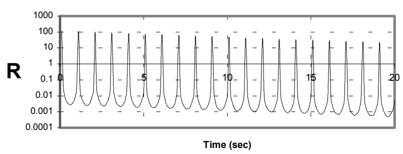

Fig. 7. Measurement error covariance with eclipsing

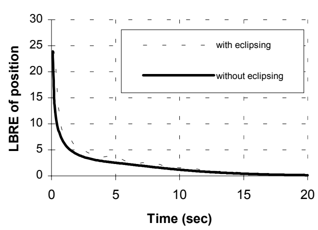

Fig. 8. Effect of Changing SNR on LBRE of Position

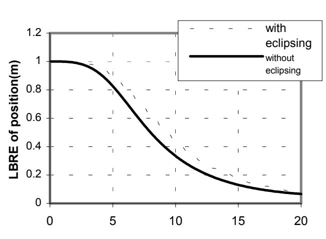

Fig.9. Effect of Changing SNR on LBRE of velocity

Thus the CRB studies address many decision problems in the early phase of the weapon system design.

The concepts have been illustrated by a fairly well known bearing only target-tracking (BOT) problem. For a typical case of high PRF MMW seeker, it has been demonstrated how the adverse effect of eclipsing may be quantified. Similarly the utility or otherwise of fusing additional signals is explored.

The example demonstrates that the CRB analysis provides a good handle for tracking system design trade off.

ACKNOWLEDGEMENT

The second author is grateful to the Aeronautics Research and Development Board, India for funding the research fellowship.

REFERENCES

[1] N.A.Shneydor, *Missile Guidance and Pursuit: Kinematics, Dynamics and Control*, Horwood Publishing, Chichester, 1998.

[2] S.Blackman and R.Popoli, *Design and Analysis of Modern Tracking Systems*, Artech House, Boston, 1999.

[3] B.Ristic, M.Sanjeev.Arulampalam, "Tracking a manoeuvring target using angle only measurement algorithm and performance", Signal Processing, Elsevier vol.83, 2003, pp1223-1238.

[4] Shovan Bhaumik, "Studies in State Estimation and Target Tracking Using Kalman Filter and Particle Filter", M.E *Dissertation*, Department of Electrical Engineering, Jadavpur University, Kolkata, India, 2004.

[5] B.Ristic, A.Farina, D.Benvenuti, M.S.Arulampalam, "Performance bounds and comparison of nonlinear filters for tracking a ballistic object on re-entry", *IEE Proc. Radar, Sonar, Navigation,* Vol. 150, No. 2, 2003, pp. 65-70.

[6] P.Tichavsky, Carlos H. Muravchik and Arye Nehorai "Posterior Cramer-Rao Bounds for Discrete-Time Non Linear filtering", *IEEE Transaction on Signal Processing*, Vol-46, No-5, 1998, pp 1386-1396.

[7] Thomas H Kerr , "Status of CR-Like Lower Bounds for Nonlinear Filtering",*IEEE Transaction on Aerospace and Electronics Systems,* vol-25,No-5, 1989, pp 590-601.

[8] Y.T Chan, T.A.Rea, "Passive Tracking Scheme for a Single Stationary Observer", IEEE Transactions on Aerospace and Electronic Systems, Vol 38, No.3, 2002, pp 1046-1054.

[9] Bar-Shalom Y, Li X, *Estimation and Tracking- Principles, Techniques and Software*, Artech House, 1993.

[10] S.Bhaumik, S.Sadhu, T.K.Ghoshal, "Cramer Rao Bounds for Bearing only Tracking", submitted to *Asian Journal of Control*, (June, 2004).

[11] S. Sadhu, S. Mondal, M. Srinivasan, T. K. Ghoshal, "Sigma Point Kalman Filter for Bearing only Tracking," submitted to *IEE Proceedings Proc. Radar, Sonar & Navigation* in 2004.

[12] Lin X, Kirubarajan T, Bar-Shalom Y, Maskell S, "Comparison of EKF, Pseudomeasurement and Particle Filters for a Bearing-only Target Tracking Problem", *Proc. SPIE Conference of Signal and Data Processing of small Targets*, Orlando, FL, 2002.

[13] P Zarchan , *Tachtical and Strategic Missile Guidance*, 4th edition, AIAA, 2002.

[14] R.N.Bhattacharyya, T.V.Rao, S.Sadhu, and T.K.Ghoshal, 'Control Structures and Properties of Missile Seekers', *Journal of Institution of Engineers (India)*, Vol 82, Pt 2, March 2002.

[15] Zrnic, B.M.; Zejak, A.J.; Simic, I.S.; "The eclipsing zone problem in the chirp radar", EUROCON'2001, Trends in Communications, International Conference on. , Volume: 2 , 4-7 July 2001, pp 329 – 332.