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Detectability and Observer Design for Linear Descriptor Systems*

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Abstract— **In this paper, a method is proposed to design Luenberger type observer for a class of linear descriptor systems satisfying complete detectability condition. The method is based on the properties of restricted system equivalent, derived here from a given descriptor system by means of simple matrix theory. Using restricted system equivalent form, equivalence between the detectability of a given descriptor system and that of a normal system has been established. Coefficient matrices of the proposed observer have been synthesized using pole placement technique of normal system theory and LMI approach based on the Lyapunov stability theory.**

I. INTRODUCTION

Descriptor systems arise in modeling of many real and practical systems, e.g. electrical network analysis, power systems, constrained mechanics, economic systems, chemical control process. Depending on the area, descriptor system is referred by variety of names, *viz.* differential algebraic equations (DAEs), singular, implicit, generalized state space, noncanonic, degenerate, semi-state and nonstandard systems. A linear time invariant descriptor system, $\sum(E^*,A^*,B^*,C)$, could be written as:

$$E^*\dot{x} = A^*x + B^*u, \qquad (1a)$$
$$y = Cx, \qquad (1b)$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, and $y \in \mathbb{R}^p$ are the state vector, the input vector and the output vector, respectively. $E^* \in \mathbb{R}^{n\times n}$ $A^* \in \mathbb{R}^{n\times n}$, $B^* \in \mathbb{R}^{n\times m}$ and $C \in \mathbb{R}^{p\times n}$ are known constant matrices, and the rank of $E^* = r < n$. Without loss of generality, we assume that rank of $B^* = m$ (full column rank) and rank of $C = p$ (full row rank). If $E \equiv I$, then the system is called normal system and is denoted by $\sum(A^*,B^*,C)$.

To design a control system, the knowledge of the states of the system is important. But it is not always possible or necessary to measure all the state variables. In such cases, the states can be estimated from the output of another dynamical system, which is called an observer for the given system. An observer is a mathematical realization which uses the input and output information of a given system and its output asymptotically approaches to the true state values of the given system.

In the normal system, literature concerned with the design of observers can be divided in three main classes: Luenberger type observers [1]–[3], sliding mode observers [4], and robust observers [5]. Luenberger in [1], [2] established the basic frame-work on which most of the state observers are based today. The difference among various type of observers lies in either required conditions on the system operators or methodologies to find the coefficient matrices of the proposed observers. Many researchers have extended these methods to design observers for descriptor system, see, e.g. [6]. The observer structure for a descriptor system may be in normal form [7]–[20] or in descriptor form [21], [22].

In this paper, we assume the following conditions on the given system (1):

(a) $rank \begin{bmatrix} E^* \\ C \end{bmatrix} = n$,

(b) $rank \begin{bmatrix} \lambda E^* - A^* \\ C \end{bmatrix} = n \ \forall \ \lambda \ \in \ \bar{\mathbb{C}}^+$.

where $\mathbb{C}$ represents the set of complex numbers. $\bar{\mathbb{C}}^+ = \{s|s \in \mathbb{C}, Re(s) \geq 0\}$ is the closed right half complex plane.

Some useful terms are defined as follows:

(1) System (1) is said to be *completely detectable* if both of the conditions (a) and (b) hold.
(2) System (1) is called *dual normalizable* if condition (a) holds.
(3) System (1) is said to be *detectable* if condition (b) holds.
(4) System (1) is said to be *R-observable* if condition (b) holds for $\forall \ \lambda \ \in \ \mathbb{C}$.
(5) System (1) is said to be *completely observable* if condition (b) holds for $\forall \ \lambda \ \in \ \mathbb{C}$ along with condition (a).

It is proved, see [23], that the above condition (b) is necessary (but not sufficient) for the existence of any Luenberger type observer for a descriptor system (1). It is also proved that if necessary condition (b) holds, observer for system (1) can be designed by applying condition (a) either directly on system (1) or a restricted system equivalent to system (1), see, e.g. [7]–[13]. In [7], a recursive algorithm, which is based on the staircase form of given system, is presented to construct an observer for a given observable descriptor system. Using Moore-Penrose inverse and Drazin inverse, Shafari *et.al.* [8] presented a design of minimal-order observer for regular and observable descriptor systems. Regularity is used in [9], [10] to derive an equivalent descriptor standard system, and then observability of derived systems is used to design an observer for the given descriptor system. Ren and Zang [11] assumed complete detectability of a given system and converted the observer design problem to an LMI problem. In [12], condition of impulse observability

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and detectability is assumed for a given system and a non-singular transformation is applied to design full and reduced order observers. [13] extends the results of [12] to the systems with unknown inputs.

In this paper equivalence between the detectability of a given descriptor system and that of normal system has been established in the form of a lemma. A new method to design full order Luenberger type observer has been proposed based on the pole placement method and LMI method of normal matrix theory. Compared to the methods available in literature the proposed method is simple and easy to understand and implement.

II. Problem Description and Design Approach

The problem is to design matrices N, L, and M of compatible dimensions such that the following normal system becomes a full order state observer for system (1), *i.e.*, $\hat{x} \to x$ as $t \to \infty$:

$$\begin{aligned} \dot{z} &= Nz + Bu + Ly, && (2a) \\ \hat{x} &= z + My. && (2b) \end{aligned}$$

Our approach is as follows:

- Under the condition of dual normalizability of given system $\sum(E^*, A^*, B^*, C)$, restricted system equivalent $\sum(E, A, B, C)$ is derived, where $E = RE^*$, $A = RA^*$, and $B = RB^*$. The existence of an invertible matrix, R, is proved in the Lemma 1.
- Under the same assumption, the Lemma 2 establishes the equivalence between the detectability of given descriptor system $\sum(E^*, A^*, B^*, C)$ and that of normal matrix pair (A, C).
- With the help of the Lemma 1 and the Lemma 2, the Theorem 1 proves that the system (2) is state observer for the system (1).

Also, we provide an alternative LMI based design method to solve the problem. The proposed method is demonstrated successfully with the help of two examples.

III. Main Results

Lemma 1: Let the given descriptor system $\sum(E^*, A^*, B^*, C)$ be dual normalizable. Then there exists a non-singular matrix $R \in \mathbb{R}^{n\times n}$ such that the given system is restricted system equivalent to the following descriptor system $\sum(E, A, B, C)$:

$$\begin{aligned} E\dot{x} &= Ax + Bu, && (3a) \\ y &= Cx, && (3b) \end{aligned}$$

where $E = RE^*$, $A = RA^*$ and $B = RB^*$.
Moreover,

$$rank \begin{bmatrix} I - E \\ C \end{bmatrix} = p. \qquad (4)$$

Proof: By the definition of restricted system equivalence, see [22], first claim is obvious. Since C is of full row rank matrix, $\exists$ a nonsingular matrix $P \in \mathbb{R}^{n\times n}$ such that $CP = [I_p, 0]$. Let $E^*P = \begin{bmatrix} E_{11} & E_{12} \\ E_{21} & E_{22} \end{bmatrix}$. Thus,

$$\begin{aligned} rank \begin{bmatrix} E^* \\ C \end{bmatrix} &= rank \begin{bmatrix} E^*P \\ CP \end{bmatrix} \\ &= rank \begin{bmatrix} E_{11} & E_{12} \\ E_{21} & E_{22} \\ I_p & 0 \end{bmatrix} = n. \end{aligned} \qquad (5)$$

It follows from (5) that

$$rank \begin{bmatrix} E_{12} \\ E_{22} \end{bmatrix} = n - p \text{ (full column rank).}$$

So $\exists$ a nonsingular matrix $R_0 \in \mathbb{R}^{n\times n}$ such that $R_0 \begin{bmatrix} E_{12} \\ E_{22} \end{bmatrix} = \begin{bmatrix} 0 \\ I_{n-p} \end{bmatrix}$. Let $R_0 \begin{bmatrix} E_{11} \\ E_{21} \end{bmatrix} = \begin{bmatrix} \tilde{E}_{11} \\ \tilde{E}_{21} \end{bmatrix}$. Then

$$R_0 \begin{bmatrix} E_{11} & E_{12} \\ E_{21} & E_{22} \end{bmatrix} = \begin{bmatrix} \tilde{E}_{11} & 0 \\ \tilde{E}_{21} & I_{n-p} \end{bmatrix} \qquad (6)$$

Equation (6) implies that

$$rank \begin{bmatrix} I - R_0E^*P \\ CP \end{bmatrix} = rank \begin{bmatrix} I - \tilde{E}_{11} & 0 \\ -\tilde{E}_{21} & 0 \\ I_p & 0 \end{bmatrix} = p.$$

If we take $PR_0 = R$ then

$$\begin{aligned} rank \begin{bmatrix} I - RE^* \\ C \end{bmatrix} &= rank \begin{bmatrix} P^{-1} & 0 \\ 0 & I_p \end{bmatrix} \begin{bmatrix} I - PR_0E^* \\ C \end{bmatrix} P \\ &= rank \begin{bmatrix} I - R_0E^*P \\ CP \end{bmatrix} = p. \end{aligned}$$

Thus the conclusion (4) holds. ■

Remark 1: Detectability of systems (1) and (3) is equivalent due to the following fact:
$rank \begin{bmatrix} \lambda E^* - A^* \\ C \end{bmatrix} = rank \begin{bmatrix} \lambda RE^* - RA^* \\ C \end{bmatrix} \ \forall\ \lambda \in \bar{\mathbb{C}}^+$.

Remark 2: For any arbitrary descriptor system $\sum(E, A, B, C)$, it can be proved that (4) implies the dual normalizability of the system (for the proof, see [24]). But above Lemma proves that if system $\sum(E^*, A^*, B^*, C)$ is dual normalizable then always we can find equivalent system $\sum(E, A, B, C)$ such that (4) is satisfied. In the design of observer, if the given system $\sum(E^*, A^*, B^*, C)$ directly satisfies a condition like (4), i.e. rank $\begin{bmatrix} I - E^* \\ C \end{bmatrix} = p$, then there is no need to calculate R. In this case we take $R = I_n$. Algorithm to find matrix R is given in the Appendix of this paper.

Before proving the next Lemma, we shall define the detectability of matrix pair (A, C). A matrix pair (A, C) is detectable iff $\exists$ K of compatible dimension such that $(A - KC)$ is a stable matrix. Moreover, matrix pair (A, C) is detectable iff $rank \begin{bmatrix} \lambda I - A \\ C \end{bmatrix} = n$ for all $\lambda \in \bar{\mathbb{C}}^+$ or $\lambda \in \sigma(A)_0^+$, where $\sigma(A)_0^+ = \{\lambda | Re(\lambda) \geq 0,\ \lambda \text{ is a eigenvalue of A}\}$.

Lemma 2: Under the assumption of Lemma 1, the following statements are equivalent.

(1) Descriptor system $\sum(E^*, A^*, B^*, C)$ is detectable.

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(2) Normal system $\sum(A,B,C)$, i.e. matrix pair (A,C), is detectable.

Proof: It is obvious that equation (4) implies the existence of $M \in \mathbb{R}^{n\times p}$ such that

$$E = I - MC. \tag{7}$$

Thus for any $\lambda \in \bar{\mathbb{C}}^{+}$, we have

$$\begin{aligned} rank\begin{bmatrix}\lambda E - A\\ C\end{bmatrix} &= rank\begin{bmatrix}\lambda(I-MC)-A\\ C\end{bmatrix}\\ &= rank\begin{bmatrix}I & -\lambda M\\ 0 & I\end{bmatrix}\begin{bmatrix}\lambda I - A\\ C\end{bmatrix}\\ &= rank\begin{bmatrix}\lambda I - A\\ C\end{bmatrix}.\end{aligned}$$

Now, Remark 1 implies the conclusion. ∎

Remark 3: In some earlier papers on the observer design problem for descriptor systems, it is very common to prove the equivalence between the detectability of the given descriptor system and that of a normal system. The uniqueness of the work lies in the fact that this normal system is made by just dropping the singular matrix E from the descriptor system (3), which is restricted system equivalent to the given descriptor system (1). Further, the nonsingular property of proposed matrix R simplifies the conclusion of detectability of the designed normal matrix pair.

Theorem 1: Let the given system (1) be completely detectable. Then there exists matrices N, L, and M of compatible dimensions such that the system (2) is observer for the system (1).

Proof: From systems (2) and (3) the error

$$\begin{aligned} e &= x - \hat{x}\\ &= x - z - MCx\\ &= (I - MC)x - z\\ &= Ex - z \end{aligned} \tag{8}$$

gives the dynamics:

$$\begin{aligned} \dot{e} &= E\dot{x} - \dot{z}\\ &= Ax + Bu - (Nz + Bu + LCx)\\ &= (A - LC)x - N(Ex - e)\\ &= Ne + (A - LC - NE)x\\ &= Ne + (A - LC - N + NMC)x\\ &= Ne. \end{aligned} \tag{9}$$

In the construction of equations (8) and (9), we have assumed the existence of matrices M, K, N, and L of compatible orders such that

$$I - MC = E \tag{10}$$

$$N = A - KC \tag{11}$$

$$K = L - NM \tag{12}$$

where N is stable. Now the problem of designing the state observer (2) is converted into the design of the matrices M, K, N, and L such that the equations (10)-(12) are satisfied with the stability of matrix N. The Lemma 1 and Lemma 2 show the existence of M such that the equation (10) is satisfied. The Lemma 2 also provides the detectability of matrix pair (A,C). So, there exists a matrix K such that the matrix N is stable, and we can find the K using pole placement technique for normal matrix pair (A,C). Finally, using equation (12) we can find the observer gain matrix L. ∎

IV. Design of Lyapunov Equation and LMI Approach

This section shows an alternative LMI approach to find matrix K (in equation (11)) such that N is a stable matrix by using the Lyapunov theory. Let the Lyapunov function be $V = e^{T}Xe$ where X is a positive definite matrix. Then using (9) and (11) we have

$$\begin{aligned} \dot{V} &= \dot{e}^{T}Xe + e^{T}X\dot{e}\\ &= e^{T}(A - KC)^{T}Xe + e^{T}X(A - KC)e\\ &= e^{T}(A^{T}X + XA - C^{T}\tilde{K}^{T} - \tilde{K}C)e \end{aligned} \tag{13}$$

where $\tilde{K} = XK$.

According to stability theory, error dynamics (9) to be asymptotically stable if there exists two matrices $\tilde{K}$ and X such that

$$X > 0 \tag{14}$$

and

$$A^{T}X + XA - C^{T}\tilde{K}^{T} - \tilde{K}C < 0 \tag{15}$$

Due to the detectability of matrix pair (A,C), it is clear that problem (14)-(15) is feasible. Numerical solution for X and $\tilde{K}$ can be found by any LMI tool box. Then by $K = X^{-1}\tilde{K}$, we can find required matrices N and L. This approach is illustrated in the Example 2.

V. Numerical Examples

Example 1: Consider (1) described by the following matrices (This example is taken from [11])

$$E^{*} = \begin{bmatrix}0.5 & -2.5 & 0\\ 3 & -3 & 4\\ 2 & -1 & 3\end{bmatrix}$$

$$A^{*} = \begin{bmatrix}-1 & 4.5 & -0.5\\ -7 & 7 & -8\\ -5 & 3 & -6\end{bmatrix}$$

$$B^{*} = \begin{bmatrix}1 & 0 & 1\\ 0 & 1 & 1\\ 1 & 0 & -1\end{bmatrix}$$

$$C = \begin{bmatrix}1 & 0 & 1\\ 0 & 1 & 1\end{bmatrix}$$

$$u = \begin{bmatrix}e^{-t}sint & 0.2sin2t & 0.2sin3t\end{bmatrix}^{T}.$$

Since $\text{rank}\begin{bmatrix}E^{*}\\ C\end{bmatrix} = 3$ and $\text{rank}\begin{bmatrix}I - E^{*}\\ C\end{bmatrix} \neq 2$, using the

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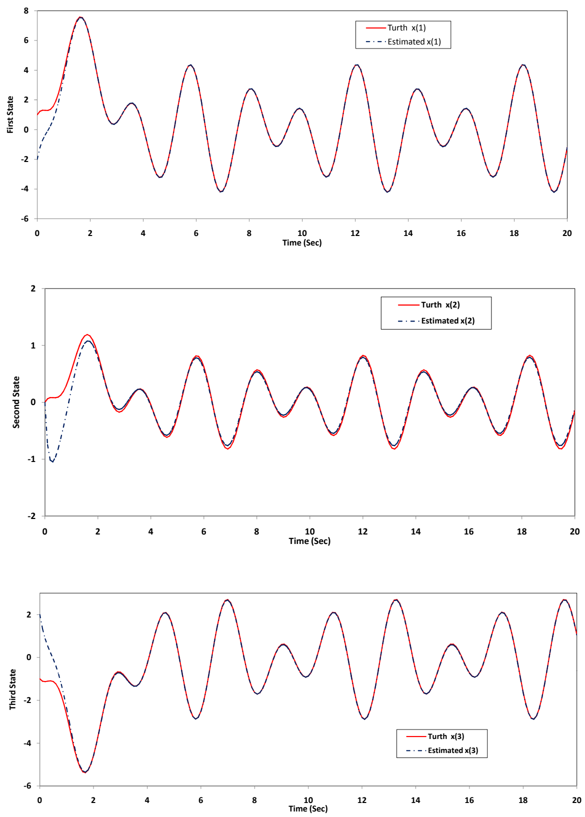

Fig. 1. Plot of truth and estimated values of states of Example 1

algorithm given in the Appendix, we calculate

$$R = \begin{bmatrix} 0.3249 & -0.0633 & -0.6983 \\ -0.0833 & -0.5000 & 0.5833 \\ 0.4916 & -0.0633 & 0.1350 \end{bmatrix}.$$

Then

$$E = \begin{bmatrix} -1.4241 & 0.0759 & -2.3481 \\ -0.3750 & 1.1250 & -0.2500 \\ 0.3259 & -1.1741 & 0.1519 \end{bmatrix}$$

$$A = \begin{bmatrix} 3.6098 & -1.0759 & 4.5338 \\ 0.6667 & -2.1250 & 0.5417 \\ -0.7236 & 2.1741 & -0.5495 \end{bmatrix}$$

$$B = \begin{bmatrix} -0.3734 & -0.0633 & 0.9599 \\ 0.5000 & -0.5000 & -1.1667 \\ 0.6266 & -0.0633 & 0.2933 \end{bmatrix}.$$

Now, we can check that rank$\begin{bmatrix} I-E \\ C \end{bmatrix} = 2$ and $M = \begin{bmatrix} 2.4241 & -0.0759 \\ 0.3750 & -0.1250 \\ -0.3259 & 1.1741 \end{bmatrix}$. Since matrix (A,C) is detectable, we calculate

$$K = \begin{bmatrix} 8.6098 & -1.0759 \\ 0.6667 & 3.8750 \\ -0.7236 & 2.1741 \end{bmatrix}$$

by using pole placement technique to shift the positive eigenvalues of matrix A in the open left half plane such that the eigenspectrum of $N = \{-5, -6, -2\}$. If we take $x(0) = \begin{bmatrix} 1 & 0 & -1 \end{bmatrix}^T$, $\hat{x}(0) = \begin{bmatrix} -2 & 0 & 2 \end{bmatrix}^T$, then the initial condition for observer is

$$z(0) = \begin{bmatrix} -2.0759 & -0.1250 & 3.1741 \end{bmatrix}^T.$$

Simulation results are plotted in the Figure 1, which reveals that the estimated values of states follow the truth states well.

Example 2: Consider (1) described by the following matrices:

$$E^* = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^* = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix}$$

$$B^* = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}^T$$

$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}.$$

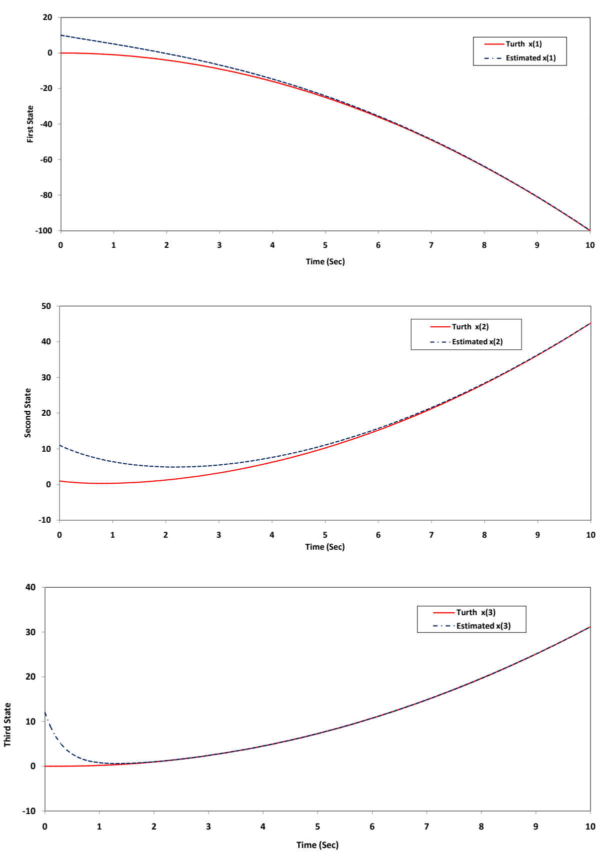

Fig. 2. Plot of truth and estimated values of states of Example 2

This system is not completely observable but completely detectable and rank$\begin{bmatrix} I-E^* \\ C \end{bmatrix} = 2$.

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Hence $R = I_3$ and $M = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$.

By using MATLAB LMI tool box we solve (14) and (15) and find $K = \begin{bmatrix} 1.5 & 0 \\ 0 & -1.5 \\ 0 & 0 \end{bmatrix}$. Thus $N = \begin{bmatrix} -.5 & 0 & 0 \\ 0 & -.5 & 0 \\ 0 & 0 & -3 \end{bmatrix}$ and $L = \begin{bmatrix} 1 & 0 \\ 0 & -1.5 \\ 0 & 0 \end{bmatrix}$.

Taking $x_0 = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}^T$, $z_0 = \begin{bmatrix} 10 & 11 & 12 \end{bmatrix}^T$, and $u = t^2$, simulation results are shown in the Figure 2.

VI. CONCLUSIONS

A Method has been developed to design a state observer for linear descriptor systems under the assumption of the complete detectability. This assumption can be verified easily from the coefficient matrices of the given system. A new restricted equivalent system which follows the same state representation as the given descriptor system, has been made with the help of an invertible matrix R. The existence and numerical algorithm of such matrix R is established.

The advantage of using this equivalent system is the fact that the detectability of its corresponding normal system is equivalent to the detectability of given descriptor system. Finally, observer design problem is converted in the solution of matrix equations (10)-(12). Solution of these equations is established by using the detectability property of normal system pair (in Theorem 1), and by using the LMI approach (in Section IV). The extension of this work to controllability and observability properties of descriptor systems and design of observers for semilinear descriptor systems is under construction.

APPENDIX

Algorithm to find the matrix R:

1. Determine
$p :=$ rank of matrix C
$n :=$order of matrix E^*.

2. Check
(i) If rank $\begin{bmatrix} I - E^* \\ C \end{bmatrix} = p$. Take $R = I_n$ and stop.
(ii) If rank $\begin{bmatrix} E^* \\ C \end{bmatrix} = n$, then go to steps 3-8.

3. Carry out the singular value decomposition (SVD) of matrix $C = U_1 \begin{bmatrix} D_1 & 0 \end{bmatrix} V_1^T$.

4. Calculate $P = V_1 \begin{bmatrix} D_1^{-1} U_1^T & 0 \\ 0 & I_{n-p} \end{bmatrix}$.

5. Calculate $\tilde{E} = E^* P \begin{bmatrix} 0 \\ I_{n-p} \end{bmatrix}$.

6. Carry out the SVD of matrix $\tilde{E} = U_2 \begin{bmatrix} D_2 \\ 0 \end{bmatrix} V_2$.

7. Calculate $R_0 = \begin{bmatrix} 0 & I_p \\ V_2^T D_2^{-1} & 0 \end{bmatrix} U_2^T$.

8. Calculate $R = P R_0$.

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