International Conference on Mathematical Sciences

PD observer design for linear descriptor systems

Mahendra Kumar Gupta[a,*], Nutan Kumar Tomar[a], Shovan Bhaumik[b]

[a]Department of Mathematics, Indian Institute of Technology Patna, India
[b]Department of Electrical Engineering, Indian Institute of Technology Patna, India

Abstract

In this paper, a method is proposed to design proportional derivative (PD) observer for linear descriptor systems satisfying complete detectability condition. The method is based on the properties of nonsingular matrix transformation, derived here from a given descriptor system. Using the result of equivalence between the detectability of a given descriptor system and that of a corresponding normal system, full order PD observer is designed. Coefficient matrices of the proposed observer have been constructed using pole placement technique and LMI approach of normal system theory. An illustrative example is given to show the effectiveness of the proposed method.

Keywords: Proportional derivative, descriptor system, observer design, linear matrix inequalities, pole-placement

1. Introduction

Descriptor systems naturally occur in a wide range of processes such as constrained mechanics [1], biological systems [2], chemical control process [3], electrical network analysis [4], environmental and economic systems [5] to name a few. Depending on the area, descriptor system is referred by variety of names, *viz.* differential algebraic equations (DAEs), singular, implicit, generalized state space, noncanonic, degenerate, semi-state and nonstandard systems. A linear time invariant descriptor system, $\sum(E, A, B, C)$, could be written as:

$$E\dot{x} = Ax + Bu, \tag{1a}$$
$$y = Cx, \tag{1b}$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, and $y \in \mathbb{R}^p$ are the state vector, the input vector and the output vector, respectively. $E,\ A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$ and $C \in \mathbb{R}^{p\times n}$ are known constant matrices, and the rank of $E = r < n$. If $E \equiv I$, then the system is called normal system and is denoted by $\sum(A, B, C)$.

An observer is a mathematical realization which uses the input and output information of a given system and its output asymptotically approaches to the true state values of the given system. Literature concerned with the design of observers for normal systems can be divided in three main classes: Luenberger type observers [6–8], sliding mode observers [9], and robust observers [10]. Luenberger in [6, 7] established the basic frame-work on which most of the state observers are based today. Many researchers have extended these methods to design full and reduced order observers for descriptor systems. The observer system for a descriptor system may be in normal form [11–19] or in descriptor form [20, 21].

Extensions of Luenberger observers have recently been done through proportional integral (PI) observers [21–26] for descriptor systems, which have the advantage of being robust to uncertainties. However, the literature on proportional derivative (PD) observers for descriptor systems is not so rich, see [27–29]. Gao in [27] developed a new parametrization of all observers for descriptor systems, based on the design of PD observers. Wu and Duan [28] designed PD observer for linear descriptor system with proportional output vector and derivative output vector. Ren and Zang [29] assumed complete detectability condition on a given descriptor system and converted the observer design problem to an LMI problem. In [30] authors proposed a type of generalized proportional-integral-derivative observers for descriptor linear systems, based on generalized Sylvester matrix equations. Ting et al. [31] designed proportional-derivative observer for normal systems with unknown inputs using descriptor system approach for non-minimum phase systems.

In this paper, we assume the following conditions on the given system (1):

(a) $rank \begin{bmatrix} E \\ C \end{bmatrix} = n,$

(b) $rank \begin{bmatrix} \lambda E - A \\ C \end{bmatrix} = n \ \forall \ \lambda \ \in \ \bar{\mathbb{C}}^{+}.$

where $\mathbb{C}$ represents the set of complex numbers. $\bar{\mathbb{C}}^{+} = \{s | s \in \mathbb{C}, Re(s) \geq 0\}$ is the closed right half complex plane.

Some useful terms are defined as follows [29]:

*Corresponding author
Email address: mahendra@iitp.ac.in (Mahendra Kumar Gupta)

[1]This work was supported by CSIR, New Delhi (Grant No. 25(0195)11/EMR-II).

ISBN - 978-93-5107-261-4

(1) System (1) is called *dual normalizable* if condition (a) holds.

(2) System (1) is said to be *detectable* if condition (b) holds.

(3) System (1) is said to be *completely detectable* if both of the conditions (a) and (b) hold.

(4) System (1) is said to be *completely observable* if condition (b) holds for $\forall\ \lambda\ \in\ \mathbb{C}$ along with condition (a).

In this paper, using the equivalence between the detectability of a given descriptor system and that of a corresponding normal system, a new method has been proposed to design full order PD observer. Stability of error dynamics of observer is achieved by pole placement method and LMI approach of normal matrix theory. Compared to methods available in literature the proposed method is simple and straightforward.

2. Problem description and design approach

The problem is to design matrices N, R, L, and M of compatible dimensions such that the following normal system becomes a full order state observer for system (1), *i.e.*, $\hat{x} \to x$ as $t \to \infty$:

$$\dot{\hat{x}} = N\hat{x} + RBu + Ly + M\dot{y} \tag{2}$$

Our approach is as follows:

- Under the condition of dual normalizability of a given system $\sum(E, A, B, C)$, restricted system equivalent $\sum(RE, RA, RB, C)$ is derived in the Lemma 1.
- Lemma 2 establishes the equivalence between the detectability of given descriptor system $\sum(E, A, B, C)$ and that of normal matrix pair (RA, C).
- With the help of the Lemma 1 and the Lemma 2, the Theorem 1 proves that the system (2) is a state observer for the given descriptor system (1).

Also, we provide an alternative LMI based design method to solve the problem. The proposed method is demonstrated successfully with the help of one example.

3. Main results

Lemma 1. *Let any matrix pair (E, C), where $E \in \mathbb{R}^{n\times n}$ and $C \in \mathbb{R}^{p\times n}$ satisfies following condition:*

$$\begin{bmatrix} E \\ C \end{bmatrix} = n$$

Then there exists a nonsingular matrix R such that

$$rank \begin{bmatrix} I - RE \\ C \end{bmatrix} = rank(C) \tag{3}$$

Proof of this Lemma can be found in [32]. Here it is notable that the aforesaid R is not unique. One numerically reliable algorithm to find such matrix R is given in the Appendix of this paper. It can be seen that following system is restricted system equivalent to the system (1)

$$\begin{aligned} RE\dot{x} &= RAx + RBu, \\ y &= Cx, \end{aligned} \tag{4}$$

Remark 1. Detectability of systems (1) and (4) is equivalent due to the following fact:
$rank \begin{bmatrix} \lambda E - A \\ C \end{bmatrix} = rank \begin{bmatrix} \lambda RE - RA \\ C \end{bmatrix}\ \forall\ \lambda\ \in\ \bar{\mathbb{C}}^{+}.$

Remark 2. If descriptor system (1) satisfies the following condition

$$rank \begin{bmatrix} I - E \\ C \end{bmatrix} = rank(C) \tag{5}$$

then it can be proved easily that system (1) is dual normalizable [33]. But above Lemma proves that if system (1) is dual normalizable then always we can find equivalent system (4) such that (3) is satisfied. In the design of observer, if the given system (1) directly satisfies condition (5), then there is no need to calculate R. In this case we take $R = I_n$.

Before proving the next Lemma, we shall define the detectability of matrix pair $(\mathcal{A}, \mathcal{C})$. A matrix pair $(\mathcal{A}, \mathcal{C})$ is detectable iff $\exists\ K$ of compatible dimension such that $(\mathcal{A} - K\mathcal{C})$ is a stable matrix. Moreover, matrix pair $(\mathcal{A}, \mathcal{C})$ is detectable iff $rank \begin{bmatrix} \lambda I - \mathcal{A} \\ \mathcal{C} \end{bmatrix} = n$ for all $\lambda\ \in\ \bar{\mathbb{C}}^{+}$ or $\lambda\ \in \sigma(\mathcal{A})_0^{+}$, where $\sigma(\mathcal{A})_0^{+} = \{\lambda | Re(\lambda) \geq 0,\ \lambda$ is a eigenvalue of $\mathcal{A}\}$.

Lemma 2. *[32] Under the assumption of Lemma 1, the following statements are equivalent.*

(1) Descriptor system $\sum(E, A, B, C)$ is detectable.

(2) Normal system $\sum(RA, RB, C)$, i.e. matrix pair (RA, C), is detectable.

Proof. It is obvious that equation (3) implies the existence of $M \in \mathbb{R}^{n\times p}$ such that

$$RE = I - MC. \tag{6}$$

Thus for any $\lambda\ \in\ \bar{\mathbb{C}}^{+}$, we have

$$\begin{aligned} rank \begin{bmatrix} \lambda RE - RA \\ C \end{bmatrix} &= rank \begin{bmatrix} \lambda(I - MC) - RA \\ C \end{bmatrix} \\ &= rank \begin{bmatrix} I & -\lambda M \\ 0 & I \end{bmatrix} \begin{bmatrix} \lambda I - RA \\ C \end{bmatrix} \\ &= rank \begin{bmatrix} \lambda I - RA \\ C \end{bmatrix}. \end{aligned}$$

Now, Remark 1 implies the conclusion.

ISBN - 978-93-5107-261-4

Theorem 1. *Let the given system (1) be completely detectable. Then there exists matrices N, L, and M of compatible dimensions such that system (2) is a observer for system (1).*

PROOF. From equations (2) and (4) the error

$$e = x - \hat{x} \tag{7}$$

gives the dynamics:

$$\begin{aligned}
\dot{e} &= \dot{x} - \dot{\hat{x}} \\
&= \dot{x} - (N\hat{x} + RBu + Ly + M\dot{y}) \\
&= \dot{x} - (N\hat{x} + RBu + Ly + MC\dot{x}) \\
&= (I_n - MC)\dot{x} - (N\hat{x} + RBu + LCx) \\
&= E\dot{x} - (N\hat{x} + RBu + LCx) \\
&= RAx + RBu - (N\hat{x} + RBu + LCx) \\
&= (RA - LC)x - N(x - e) \\
&= Ne + (RA - LC - N)x \\
&= Ne.
\end{aligned} \tag{8}$$

In the construction of equations (8), we have assumed the existence of matrices M, L, and N of compatible dimensions such that

$$I - MC = E \tag{9}$$

$$N = RA - LC \tag{10}$$

where N is stable. Now the problem of designing the state observer (2) is converted into the design of the matrices M, N, and L such that the equations (9)-(10) are satisfied. The Lemma 1 and Lemma 2 show the existence of M such that the equation (9) is satisfied. The Lemma 2 also provides the detectability of matrix pair (RA, C). So, there exists a matrix L such that the matrix N is stable, and we can find the L using pole placement technique for normal matrix pair (RA, C).

4. Design of Lyapunov equation and LMI approach

This section shows an alternative LMI approach to find matrix L (in equation (10)) such that N is a stable matrix by using the Lyapunov theory. Let the Lyapunov function be $V = e^T Pe$ where P is a positive definite matrix. Then using (8) and (10) we have

$$\begin{aligned}
\dot{V} &= \dot{e}^T Pe + e^T P\dot{e} \\
&= e^T (RA - LC)^T Pe + e^T P(RA - LC)e \\
&= e^T (A^T R^T P + PRA - C^T \tilde{L}^T - \tilde{L}C)e
\end{aligned} \tag{11}$$

where $\tilde{L} = PL$.

According to stability theory, error dynamics (8) to be asymptotically stable if there exists two matrices $\tilde{L}$ and P such that

$$P > 0 \tag{12}$$

and

$$A^T R^T P + PRA - C^T \tilde{L}^T - \tilde{L}C < 0 \tag{13}$$

Due to the detectability of matrix pair (RA, C), it is clear that problem (12)-(13) is feasible. Numerical solution for P and $\tilde{L}$ can be found by any LMI tool box. Then by $L = P^{-1}\tilde{L}$, we can calculate matrix N. This approach is illustrated in Example 1.

5. Numerical example

Example 1. Consider the descriptor system (1) described by the following matrices:

$$E = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix},$$

$$B = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}^T, C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}.$$

This system is not completely observable but completely detectable and rank $\begin{bmatrix} I - E \\ C \end{bmatrix} = 2$.

Hence $R = I_3$ and $M = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$.

By using MATLAB LMI tool box we solve (12) and (13) and find $L = \begin{bmatrix} 1.5 & 0 \\ 0 & -1.5 \\ 0 & 0 \end{bmatrix}$. Thus $N = \begin{bmatrix} -.5 & 0 & 0 \\ 0 & -.5 & 0 \\ 0 & 0 & -3 \end{bmatrix}$

Taking $x_0 = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}^T$, $z_0 = \begin{bmatrix} 10 & 11 & 12 \end{bmatrix}^T$, and $u = t^2$, simulation results are shown in Figure 1.

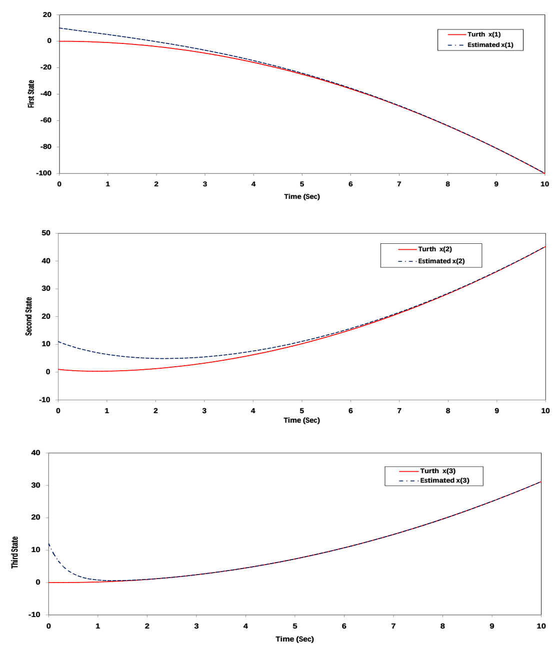

Figure 1: Plot of truth and estimated values of states of Example 1

ISBN - 978-93-5107-261-4

6. CONCLUSIONS

A Method has been developed to design a proportional derivative observer for square linear descriptor systems under the assumption of the complete detectability. Equivalence of detectability of the given descriptor system and its corresponding normal system is used for deriving results. Importance of this work is in the fact that, this normal system is made by only removing singular matrix RE, without changing in any other system equation. Finally, observer design problem is converted in the solution of matrix equations (9)-(10). Solution of these equations is established by using the detectability property of normal system pair, and by using the LMI approach. The extension of this work to design of proportional derivative observers for semilinear descriptor systems is under study.

APPENDIX

Algorithm to find the matrix R:

1. Determine
$p :=$ rank of matrix C
$n :=$order of matrix E.

2. Check
(i) If rank $\begin{bmatrix} I-E \\ C \end{bmatrix} = p$. Take $R = I_n$ and stop.
(ii) If rank $\begin{bmatrix} E \\ C \end{bmatrix} = n$, then go to steps 3-8.

3. Carry out the singular value decomposition (SVD) of matrix $C = U_1 \begin{bmatrix} D_1 & 0 \end{bmatrix} V_1^T$.

4. Calculate $P = V_1 \begin{bmatrix} D_1^{-1}U_1^T & 0 \\ 0 & I_{n-p} \end{bmatrix}$.

5. Calculate $\tilde{E} = EP \begin{bmatrix} 0 \\ I_{n-p} \end{bmatrix}$.

6. Carry out the SVD of matrix $\tilde{E} = U_2 \begin{bmatrix} D_2 \\ 0 \end{bmatrix} V_2$.

7. Calculate $R_0 = \begin{bmatrix} 0 & I_p \\ V_2^T D_2^{-1} & 0 \end{bmatrix} U_2^T$.

8. Calculate $R = PR_0$.

References

[1] S. J. Small. *Runge-Kutta type methods for differential-algebraic equations in mechanics*. University of Iowa, 2011.
[2] Q. Zhang, C. Liu and X. Zhang. *Complexity, analysis and control of singular biological systems*. Springer, 2012.
[3] A. Kumar and P. Daoutidis. *Control of nonlinear differential algebraic equation systems with applications to chemical processes*. CRC Press, 1999.
[4] R. Riaza. *Differential-algebraic systems: Analytical aspects and circuit applications*. World Scientific, 2008.
[5] D. G. Luenberger and A. Arbel. Econometrica: Journal of the Econometric Society (1977) 991.
[6] D. G. Luenberger. IEEE Trans. Military Electronics 8(2) (1964) 74.
[7] D. G. Luenberger. IEEE Trans. Autom. Control 11(2) (1966) 190.
[8] D. G. Luenberger. IEEE Trans. Autom. Control 16(6) (1971) 596.
[9] S. K. Spurgeon. Int. J. Systems Science 39(8) (2008) 751.
[10] R. L. Carroll and D. Lindorff. IEEE Trans. Autom. Control 18(5) (1973) 428.
[11] M. Verhaegen and P. Van Dooren. Syst. Control Lett. 8(1) (1986) 29.
[12] B. Shafai and R. Carroll. Int. J. Control 45(3) (1987) 1075.
[13] N. Minamide, N. Arii and Y. Uetake. Int. J. Control 50(6) (1989) 2141.
[14] Y. Uetake. Int. J. Control 50(1) (1989) 89.
[15] M. Darouach and M. Boutayeb. IEEE Trans. Autom. Control 40(7) (1995) 1323.
[16] P. C. Müller and M. Hou. IEEE Trans. Autom. Control 38(11) (1993) 1666.
[17] C.-W. Yang and H.-L. Tan. Int. J. Control 49(6) (1989) 1937.
[18] F. Lewis. Automatica 26(2) (1990) 411.
[19] M. El-Tohami, V. Lovass-Nagy and R. Mukundan. Int. J. Control 46(3) (1987) 841.
[20] G.-R. Duan. *Analysis and design of descriptor linear systems* . Springer, 2010.
[21] A.-G. Wu and G.-R. Duan. IEEE Trans. Syst. Man Cybern. Part B Cybern. 36(6) (2006) 1423.
[22] H. S. Kim, T. K. Yeu and S. Kawaji. Trans. Control, Autom. Syst. Eng. 3(2) (2001) 77.
[23] D. Koenig and S. Mammar. IEEE Trans. Autom. Control 47(12) (2002) 2057.
[24] A.-G. Wu and G.-R. Duan. IEEE Trans. Circuits Syst. Regul. Pap. 53(12) (2006) 282.
[25] Z. Gao and S. X. Ding. IET Control Theory Appl. 1(5) (2007) 1208.
[26] A.-G. Wu, G.-R. Duan and W. Liu. Asian J. Control 14(2) (2012) 476.
[27] Z. Gao. J. Franklin Institute 342(5) (2005) 551.
[28] A.-G. Wu and G.-R. Duan. Int. J. Control Autom. Syst. 5(1) (2007) 93.
[29] J. Ren and Q. Zhang. Int. J. Control Autom. Syst. 8(4) (2010) 735.
[30] A.-G. Wu, G.-R. Duan and Y.-M. Fu. IEEE Trans. Syst. Man Cybern. Part B Cybern. 37(5) (2007) 1390.
[31] H.-C. Ting, J.-L. Chang and Y.-P. Chen. Int. J. Control Autom. Syst. 9(5) (2011) 850.
[32] M. K. Gupta, N. K. Tomar and S. Bhaumik. Detectability and observer design for linear descriptor system. In *22nd Mediterranean Conference on Control and Automation* 2014, (Accepted). IEEE.
[33] M. K. Gupta, N. K. Tomar and S. Bhaumik. On Observability of Irregular Descriptor Systems. In *Advances in Control and Optimization of Dynamical Systems* 2014 376.

ISBN - 978-93-5107-261-4