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Particle-method-based formulation of risk-sensitive filter

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ABSTRACT

A novel particle implementation of risk-sensitive filters (RSF) for nonlinear, non-Gaussian state-space models is presented. Though the formulation of RSFs and its properties like robustness in the presence of parametric uncertainties are known for sometime, closed-form expressions for such filters are available only for a very limited class of models including finite state-space Markov chains and linear Gaussian models. The proposed particle filter-based implementations are based on a probabilistic re-interpretation of the RSF recursions. Accuracy of these filtering algorithms can be enhanced by choosing adequate number of random sample points called particles. These algorithms significantly extend the range of practical applications of risk-sensitive techniques and may also be used to benchmark other approximate filters, whose generic limitations are discussed. Appropriate choice of proposal density is suggested. Simulation results demonstrate the performance of the proposed algorithms.

1. Introduction

The theory of risk-sensitive filters (RSF) [1–3] had been known for over a decade, and its robustness (in a restricted but formalized sense) against model uncertainty for general nonlinear and non-Gaussian signal models has been derived in [1]. RSFs minimize the expected value of an exponential of a convex function of the estimation error and a designer-chosen parameter, called the risk-sensitive parameter. The risk-sensitive parameter provides a tool for design tradeoff [4] between the filtering performance for the nominal model and the robustness to model uncertainty. A general framework for recursive computation of the RSF problem is provided in [1], but its direct implementation, except for trivial (say linear Gaussian and finite state-space Markov chains) cases, becomes nearly impossible, as it involves intractable integrals. Approximate methods based on local linearisation (like extended Kalman filter (EKF) [3]) and linear regression filters [4] have been proposed. However, these approximate RSFs require the noise to be Gaussian and are also susceptible to track loss [4]. In this paper, we develop a sequential Monte Carlo (SMC) implementation of the RSF recursion. SMC or particle filter (PF) formalism is traditionally based on probability density functions (probability measures). The RSF recursion, however, is based on information state and the PF approach cannot be applied directly. In this proposed framework, the distribution associated with the information state is normalized so that an artificial probabilistic interpretation of the RSF may be obtained. The resulting probability measures are then approximated by a collection of random samples named particles which are propagated in time using importance sampling and re-sampling mechanisms. This particle implementation of risk-sensitive filtering does not rely on any linearity or Gaussian assumption and can be used for any model, significantly broadening the range of practical applications of RSFs. Unlike the earlier approximate RSFs, the accuracy of estimation in the proposed algorithm can be enhanced by choosing adequate number

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of particles. The proposed algorithm would, therefore, be useful in (performance) benchmarking for other approximate risk-sensitive filtering algorithms [3,4]. To the best of our knowledge, particle methods for risk-sensitive filtering have never appeared before in refereed publications. Despite its title, Ref. [5] addresses a completely different problem.

2. Formulation of the risk-sensitive estimation (RSE) problem

Let us consider the following nonlinear non-Gaussian evolution and observation equations:

$$x_{k+1} = f(x_k) + w_k \tag{1}$$

$$y_k = h(x_k) + v_k \tag{2}$$

where $x_k \in R^n$ denotes the state and $y_k \in R^p$ denotes the measurement at any instance k, where $k = 0,1,2,3,\ldots, n_{\text{step}}$, $w_k \in R^n$, $v_k \in R^p$ are additive, independent noises with known statistics. The vectors $f(x_k)$ and $h(x_k)$ are general (without any assumption of smoothness) nonlinear functions of x_k and k.

The generalized cost function for risk-sensitive estimator at any instance k can be defined as [1]

$$J_{RS}(\varsigma, k) = E\left[\exp\left(\mu_1 \sum_{i=0}^{k-1} \rho_1(\phi(x_i) - \hat{\Phi}_i) + \mu_2 \rho_2(\phi(x_k) - \varsigma)\right)\right] \tag{3}$$

The constant parameters μ_1 and μ_2 are called *risk-sensitive* parameters and ϕ is a real valued measurable function on R^n. The optimal estimate of the function ϕ at any instance i is denoted by $\hat{\Phi}_i$, and ς is a parameter. Functions $\rho_1(\cdot)$ and $\rho_2(\cdot)$ are both strictly convex, continuous and bounded from below, attaining global minima at 0.

The initial state x_0 is independent of the noise processes mentioned above and has a known probability density distribution $p(x_0) = p_0(x_0)$. The risk-sensitive cost function as above includes an accumulated error cost up to time k, with a relative weight μ_1 and the current cost weighted by μ_2.

The optimal estimate of ϕ at the current instant, denoted by $\hat{\Phi}_k$, is obtained by finding the optimum value of ς, which minimizes $J_{RS}(\varsigma, k)$, i.e.:

$$\hat{\Phi}_k = \underset{\varsigma \in R}{\operatorname{argmin}} J_{RS}(\varsigma, k) \tag{4}$$

3. Solution of the risk-sensitive estimation problem

In this and subsequent sections, the solution of the risk-sensitive filtering problem has been provided for both the predicted RSF estimate and filtered RSF estimate. The predicted RSF estimate, which has been referred to as the one-stage delay estimate by some authors [2,3], provides the a-priori estimate and the filtered RSF estimate, which has been referred to as the current information estimate [2,3], provides the a-posteriori estimate. It is straightforward to show that both the a-priori and a-posteriori estimators reduce to respective risk-sensitive Kalman filter (RSKF) for the linear Gaussian case. Note that in subsequent sections, some symbols have been re-used for predicted and filtered estimation.

3.1. Predicted RSF estimate

The solution to the RSE problem for a-priori estimation may be obtained from the following recursive relation [1] using an information state $\alpha_k(x_k)$ and probability density functions $p(x_{k+1}|x_k)$ and $p(y_k|x_k)$.

$$\alpha_k(x_k) = \int p(x_k|x_{k-1}) p(y_{k-1}|x_{k-1}) \times \exp(\mu_1 \rho_1(\phi(x_{k-1}) - \hat{\Phi}_{k-1|k-2})) \alpha_{k-1}(x_{k-1})\, dx_{k-1} \tag{5}$$

and the optimal estimate is

$$\hat{\Phi}_{k|k-1} = \underset{\varsigma \in R}{\operatorname{argmin}} \int_{-\infty}^{+\infty} \exp(\mu_2 \rho_2(\phi(x_k) - \varsigma)) \alpha_k(x_k)\, dx_k \tag{6}$$

3.2. Filtered RSF estimate

Similarly, the solution of the RSE problem for a-posteriori estimation may be obtained as

$$\alpha_k(x_k) = p(y_k|x_k) \int p(x_k|x_{k-1}) \exp(\mu_1 \rho_1(\phi(x_{k-1}) - \hat{\Phi}_{k-1|k-1})) \alpha_{k-1}(x_{k-1}) dx_{k-1} \tag{7}$$

and the optimal estimate is

$$\hat{\Phi}_{k|k} = \underset{\varsigma \in R}{\operatorname{argmin}} \int_{-\infty}^{+\infty} \exp(\mu_2 \rho_2(\phi(x_k) - \varsigma)) \alpha_k(x_k)\, dx_k \tag{8}$$

4. Probabilistic interpretation

Solution to the risk-sensitive filtering problem involves integrations defined in Eqs. (5)–(8) above. However, these integrations cannot be evaluated in closed form for general nonlinear systems. The authors propose a PF [6]-based algorithm to remove the above difficulty. It should be pointed out that the algorithms for PF cannot be applied directly and substantial modifications have to be made. The following relations provide a necessary background for the so-named risk-sensitive particle filter (RSPF).

To facilitate conversion of the above relations in PF form, we normalize the positive measure $\alpha_k(\cdot)$ and also introduce an intermediate measure $\beta_k(\cdot)$ and its normalized form.

The normalized form of $\alpha_k(\cdot)$ is given by

$$\bar{\alpha}_k(x_k) = \frac{\alpha_k(x_k)}{\int \alpha_k(x_k)\, dx_k}$$

4.1. Predicted RSF estimate

The positive measure $\alpha_k(x_k)$ is given by Eq. (5).

We define the intermediate measure:

$$\beta_k(x_k) = p(y_k|x_k)\exp(\mu_1\rho_1(\phi(x_k) - \hat{\Phi}_{k|k-1}))\alpha_k(x_k) \tag{9}$$

and its normalized version:

$$\bar{\beta}_k(x_k) = \frac{\beta_k(x_k)}{\int \beta_k(x_k)\,\mathrm{d}x_k}$$

Using such notation, we can rewrite the predicted RSF estimate as follows:

$$\bar{\alpha}_k(x_k) = \int p(x_k|x_{k-1})\bar{\beta}_{k-1}(x_{k-1})\,\mathrm{d}x_{k-1} \tag{10}$$

and

$$\hat{\Phi}_{k|k-1} = \underset{\varsigma \in R}{argmin} \int_{-\infty}^{+\infty} \exp(\mu_2\rho_2(\phi(x_k) - \varsigma))\bar{\alpha}_k(x_k)\,\mathrm{d}x_k \tag{11}$$

4.2. Filtered RSF estimate

Analogously, for the filtered RSF estimate, $\alpha_k(x_k)$ is given by Eq. (7).

The intermediate measure and its normalized form are given by

$$\beta_{k-1}(x_{k-1}) = p(y_k|x_k)\exp(\mu_1\rho_1(\phi(x_{k-1}) - \hat{\Phi}_{k-1|k-1}))\alpha_{k-1}(x_{k-1})$$

and

$$\bar{\beta}_{k-1}(x_{k-1}) = \frac{\beta_{k-1}(x_{k-1})}{\int \beta_{k-1}(x_{k-1})\,\mathrm{d}x_{k-1}}$$

Using such notation, we can rewrite the filtered RSF estimator as follows:

$$\bar{\alpha}_k(x_k) = \int p(x_k|x_{k-1})\bar{\beta}_{k-1}(x_{k-1})\,\mathrm{d}x_{k-1} \tag{12}$$

and

$$\hat{\Phi}_{k|k} = \underset{\varsigma \in R}{argmin} \int_{-\infty}^{+\infty} \exp(\mu_2\rho_2(\phi(x_k) - \varsigma))\bar{\alpha}_k(x_k)\,\mathrm{d}x_k \tag{13}$$

5. Particle implementation

The particle implementation described below follows from the normalized measures discussed in the previous section.

5.1. Predicted RSF estimate

- *Initialization*; $k = 0$
  Sample $X_0^{(i)} \sim p_0(\cdot)$ to obtain

  $$\hat{\alpha}(x_0) = \frac{1}{N}\sum_{i=1}^{N} \delta(x_0 - X_0^{(i)}), \tag{14}$$

  where N is the number of particles and i is the particle index,
  and set

  $$\hat{\Phi}_0 = \underset{\varsigma \in R}{argmin} \sum_{i=1}^{N} \exp(\mu_2\rho_2(\phi(X_0^{(i)}) - \varsigma)). \tag{15}$$

  Sample $\tilde{X}_0^{(i)} \sim q_0(\cdot)$, where $q_0(\cdot)$ is a proposal density, and compute

  $$\tilde{W}_0^{(i)} \propto \frac{p(y_0|\tilde{X}_0^{(i)})\exp(\mu_1\rho_1(\phi(X_0^{(i)}) - \hat{\Phi}_0))p_0(\tilde{X}_0^{(i)})}{q_0(\tilde{X}_0^{(i)})} \tag{16}$$

  Normalize weights such that

  $$\sum_{i=1}^{N} \tilde{W}_0^{(i)} = 1. \tag{17}$$

  Resample particles $\{\tilde{X}_0^{(i)}, \tilde{W}_0^{(i)}\}$
- *At time* k; $k > 0$
  Sample $X_k^{(i)} \sim p(.|\tilde{X}_{k-1}^{(i)})$.
  It follows that

  $$\hat{\alpha}_k(x_k) = \frac{1}{N}\sum_{i=1}^{N} \delta(x_k - X_k^{(i)}) \tag{18}$$

  and set

  $$\hat{\Phi}_{k|k-1} = \underset{\varsigma \in R}{argmin} \sum_{i=1}^{N} \exp(\mu_2\rho_2(\phi(X_k^{(i)}) - \varsigma)). \tag{19}$$

  Sample $\tilde{X}_k^{(i)} \sim q_k(.|\tilde{X}_{k-1}^{(i)})$, where $q_k(\cdot)$ is a proposal density,
  and compute weight

  $$\tilde{W}_k^{(i)} \propto \frac{p(\tilde{X}_k^{(i)}|\tilde{X}_{k-1}^{(i)})p(y_k|\tilde{X}_k^{(i)})\exp(\mu_1\rho_1(\phi(\tilde{X}_k^{(i)}) - \hat{\Phi}_{k|k-1}))}{q_k(\tilde{X}_k^{(i)}|\tilde{X}_{k-1}^{(i)})}. \tag{20}$$

  Normalize weights such that

  $$\sum_{i=1}^{N} \tilde{W}_k^{(i)} = 1. \tag{21}$$

  Resample particles $\{\tilde{X}_k^{(i)}, \tilde{W}_k^{(i)}\}$.

In this algorithm we need to select the importance distributions $q_0(x_0)$ and $q_k(x_k|x_{k-1})$. It is straightforward to check that the distribution minimizing the conditional variance of the weights is

$$q_0^{\mathrm{opt}}(x_0) \propto p(x_0)p(y_0|x_0)\exp(\mu_1\rho_1(\phi(x_0) - \hat{\Phi}_0)) \quad \text{at } k = 0 \tag{22}$$

and

$$q_k^{\mathrm{opt}}(x_k|x_{k-1}) \propto p(x_k|x_{k-1})p(y_k|x_k)\exp(\mu_1\rho_1(\phi(x_k) - \hat{\Phi}_{k|k-1})) \quad \text{at } k > 0. \tag{23}$$

It is possible to sample from these optimal importance distributions in some particular cases. If it is not possible then some approximations (sub-optimal proposals) have to be used. Choice of proposal distributions for some simple cases has been discussed in Section 6.

5.2. Filtered RSF estimate

The particle implementation for a-posteriori estimation has been derived in a similar manner, but is not being presented in this paper.

6. Case studies

Though a rigorous convergence proof of the algorithm is currently under study, we have experimentally verified the convergence of these schemes for a wide variety of problems. For a linear Gaussian system, where an exact solution using RSKF is available, it has been demonstrated by the present authors [7] that for a reasonable number of particles, the RSPF converges to the correct estimate. Convergence for a nonlinear problem has also been demonstrated in the same reference. Further empirical studies for a wide variety of problems carried out so far confirm the convergence wherever the approximate (linearized Gaussian) filters converge, without exception. The RSPF converged even for some cases where the extended risk-sensitive filter (ERSF) [3] failed to track (see case studies below and [4]).

The empirical study of convergence is carried out with the help of a measure of dispersion (variance) of the estimated value using Monte Carlo (MC) simulation (see Figs. 1 and 2). Convergence of the algorithm is empirically established when such variance asymptotically tends to zero with increase in the number of particles.

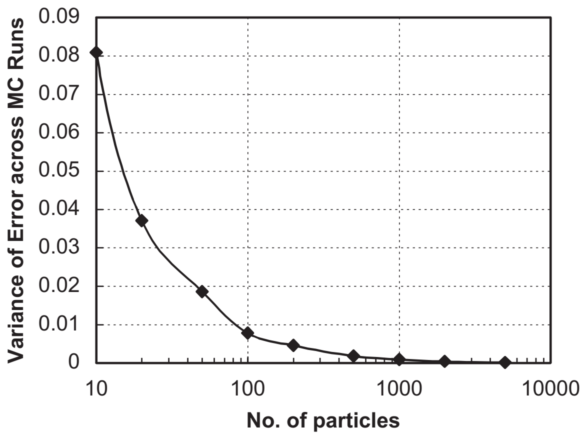

Fig. 1. Convergence of RSPF for 1D nonlinear system and measurements with Gaussian noise.

In all the case studies that follow, it has been assumed that $\rho_1(\cdot)$ and $\rho_2(\cdot)$ are quadratic and $\phi(x_k) = x_k$.

Example 1. *Convergence of RSPF for one-dimensional* (1D) *nonlinear system and linear measurements.*

The process equation is given by Eq. (1), where $w_k \sim N(0, Q_k)$, and measurement equation is

$$y_k = Hx_k + v_k, \quad \text{where } v_k \sim N(0, R_k). \tag{24}$$

Also, $Q_k = 1$, $R_k = 1$, $f(x_k) = 2 - 0.001 x_k \text{sgn}(x_k)$, $H = 1$, $x_0 = 0$ and $\mu_1 = \mu_2 = 0.2$. Fig. 1 shows that the variance of the error across MC runs converges to zero as the number of particles is increased. This indicates that, for this example, the RSPF converges as the number of particles is increased.

Example 2. *Convergence of RSPF for two-dimensional* (2D) *bearing only tracking (BOT) problem with linear process and nonlinear measurements.*

This example concerns a practical application where a target is tracked using only bearing information. This is useful for underwater tracking as well as in electronic warfare, where a surface ship may be tracked by an aircraft radar in the passive mode [9–11]. In the BOT problem, conventional filters like EKF are known to lose track. Further, in the naval scenario (due to low speed of target and low sampling rate), increased computational load of PF is unlikely to create implementation problems.

The BOT problem briefly described below has a linear process model and nonlinear measurements. A more detailed discussion on problem formulation and filter initialization may be found in [9,10].

The BOT problem [9,10] has two components, namely, the target kinematics and the tracking platform kinematics. The tracking platform moves approximately parallel to the target with approximately constant velocity, given by the following discrete time equations:

$$x_p(k) = \bar{x}_p(k) + \Delta x_p(k) \quad k = 0, 1, \ldots, n_{step} \tag{25}$$

$$y_p(k) = \bar{y}_p(k) + \Delta y_p(k) \quad k = 0, 1, \ldots, n_{step} \tag{26}$$

where $\bar{x}_p(k)$ and $\bar{y}_p(k)$ are the (known) average platform position co-ordinates, $\Delta x_p(k) \sim N(0, T^2)$ and $\Delta y_p(k) \sim N(0, T^2)$ are the mutually independent white noise sequences,

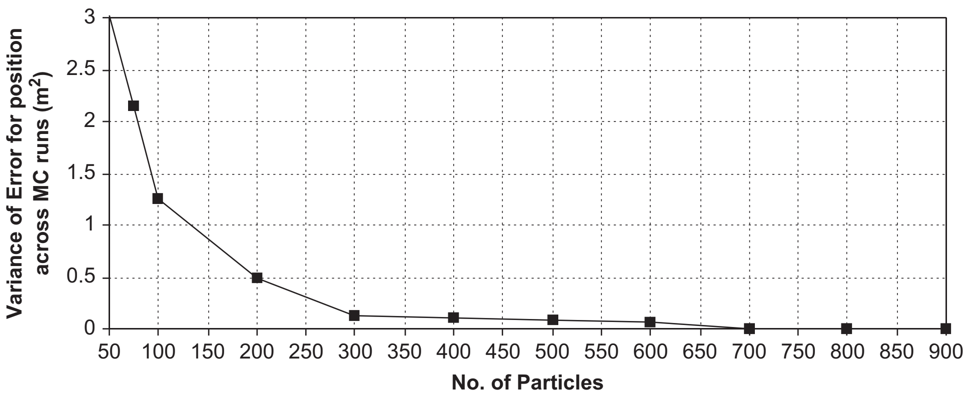

Fig. 2. Convergence of RMS error for position in 2D BOT problem.

where T is the sampling time and has a nominal value of 1 s. The mean positions of the platform are

$\bar{x}_p(k) = 4kT$ and $\bar{y}_p(k) = 20$

The target motion in the x-direction is given by

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} T^2/2 \\ T \end{bmatrix} w(k) \qquad (27)$$

where $x_1(k)$ is the position along the x-axis in meters, $x_2(k)$ is the velocity in m/s and $w(k) \sim N(0, q)$. The (unknown) true initial condition and the known noise variance are

$$\underline{x}(0) = \begin{bmatrix} 80 \\ 1 \end{bmatrix}, \quad q = 0.01\,\text{m}^2/\text{s}^4$$

The measurement equation (in bearing coordinate) is given as

$$z_m(k) = \tan^{-1} \frac{y_p(k)}{x_1(k) - x_p(k)} + v_s(k) \qquad (28)$$

where $v_s(k) \sim N(0, r_s)$ and $r_s = (3^\circ)^2$.

The effect of platform motion noises have been approximated as additive noise by using the method described in [9,10].

The initial velocity estimate for the filter is selected as $\hat{x}_2(0) = 0$ and associated variance as $P_{22}(0) = 1$. The off diagonal terms $P_{12}(0)$ and $P_{21}(0)$ are taken as zero.

Fig. 2 shows the plot of the variance of error for position (metre-squared i.e., m^2) across several MC runs at a particular time step, (i.e. 18 s), versus the number of particles. The RSPF converges with the increase in the number of particles for this nonlinear application example. The variance of error for velocity also shows a similar trend (but has not been included in this paper).

Fig. 3 shows the RMS error of position for this problem. The RMS error has been calculated over 1000 MC runs and the RSPF has been implemented with 900 particles. It is observed that at 18 s, the RMS position error for RSPF is 1.90 m, and the corresponding CRLB [8] value is 1.53 m. This time instant is chosen for comparison because at this time instant the effects of possible initial condition mismatch would be small.

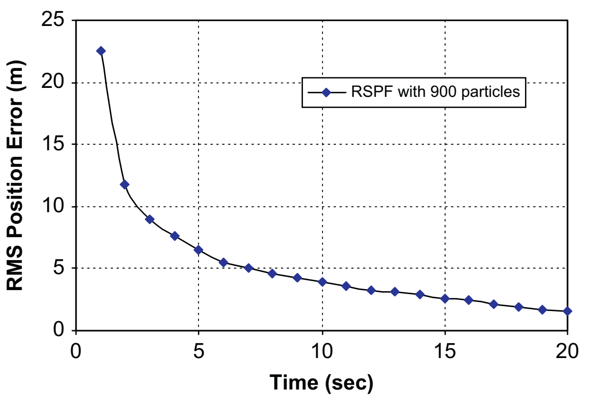

Fig. 3. RMS error for position over 1000 Monte Carlo runs in 2D BOT problem.

Example 3. *Comparative results for ERSF, central difference risk-sensitive filter (CDRSF) [4] and RSPF for* 1D *severely nonlinear plant.*

The performances of the ERSF, CDRSF and the RSPF have been studied with the following severely nonlinear problem:

$$x_{k+1} = x_k + \phi(x_k) + w_k$$
$$\text{where } \phi(\zeta) = 0.05\zeta(1 - \zeta^2) \quad w_k \sim N(0, 0.0025)$$

$$y_k = \gamma(x_k) + v_k$$
$$\text{where } \gamma(\zeta) = 0.01\zeta(1 - 1.5\zeta), \quad v_k \sim N(0, 0.0001)$$

The initial conditions of the filter and the initial error covariance are set, respectively, at 0 and 2. The initialization of the plant is random.

Fig. 4 shows a comparison of the RMS error settling performance over 1000 MC runs of the ERSF, CDRSF and RSPF. From the figure, it is observed that the RMS error of the RSPF is the lowest, followed by the CDRSF and then the ERSF for this particular problem.

However, admittedly, the computational time for the RSPF is much higher (about 200 times more) compared to the CDRSF and ERSF.

Example 4. *Computation of optimal proposal for RSPF with nonlinear system and linear measurements.*

One of the key factors for successful implementation of particle-based methods is the computation of a suitable proposal distribution. The same would be true for the PF-based risk-sensitive estimator being proposed in this paper. In this example and the next one a suitable proposal has been proposed for classes of problems using the methods suggested in [6]. This example deals with a general nonlinear system and linear measurements. The process equation is given by Eq. (1) and the measurement equation is given by Eq. (24). The optimal distribution [6] for predicted RSF estimator can be computed as $q_k^{\text{opt}}(x_k|x_{k-1}) \sim N(m_k, \Sigma_k)$ where

$$\sum\nolimits_k^{-1} = H^T R_k^{-1} H + Q_k^{-1} - 2\mu_1 I$$

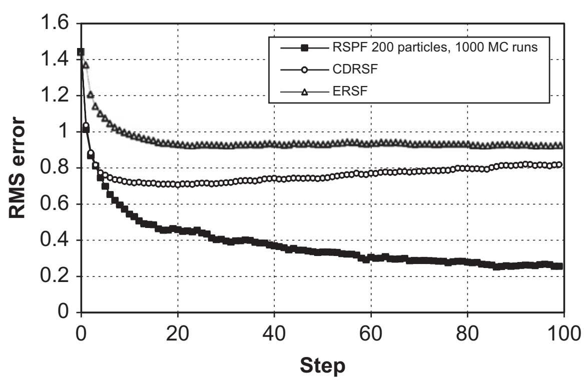

Fig. 4. RMS error settling performance of the ERSF, CDRSF and RSPF over 1000 Monte Carlo runs.

and

$$m_k = \sum_k (H^T R_k^{-1} y_k + Q_k^{-1} f(x_{k-1}) - 2\mu_1 \hat{\Phi}_{k|k-1}) \tag{29}$$

Example 5. *Computation of approximate optimal proposal with nonlinear system and measurements.*

This example deals with a general system with nonlinear process and measurements. The system dynamics and measurement equation are given by Eqs. (1) and (2), respectively, where $w_k \sim N(0, Q_k)$ and $v_k \sim N(0, R_k)$. The optimal importance distribution [6] may be approximated using local linearization.

For predicted RSF estimator, $q_k^{\text{opt}}(x_k|x_{k-1}) \sim N(m_k, \Sigma_k)$, where

$$\sum\nolimits_k^{-1} = Q_k^{-1} + \left(\left(\frac{\partial h(x_k)}{\partial x_k} \right) \Bigg|_{x_k = f(x_{k-1})} \right)^T R_k^{-1} \left(\frac{\partial h(x_k)}{\partial x_k} \right) \Bigg|_{x_k = f(x_{k-1})} - 2\mu_1 I,$$

$$m_k = \sum\nolimits_k \Bigg(Q_k^{-1} f(x_{k-1}) + \left(\left(\frac{\partial h(x_k)}{\partial x_k} \right) \Bigg|_{x_k = f(x_{k-1})} \right)^{\mathrm{T}} \times R_k^{-1} \left(y_k - h(f(x_{k-1})) + \left(\frac{\partial h(x_k)}{\partial x_k} \right) \Bigg|_{x_k = f(x_{k-1})} f(x_{k-1}) \right) - 2\mu_1 \hat{\Phi}_{(k|k-1)}$$

7. Discussion and conclusion

RSPF algorithms have been proposed. The method significantly extends the range of applications of risk-sensitive techniques to nonlinear non-Gaussian problems. Performance of the proposed algorithm has been exemplified and is compared with other approximate filters. The empirical study indicates that the RSPF converges quickly with adequate number of particles when a good proposal density is chosen.

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