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–Brief Paper–

ROBUST ENSEMBLE KALMAN FILTER BASED ON EXPONENTIAL COST FUNCTION

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ABSTRACT

This paper provides an alternative point of view to the robust estimation technique for nonlinear non Gaussian systems based on exponential quadratic cost function. The proposed method, named the risk sensitive ensemble Kalman filter (RSEnKF), is based on the ensemble Kalman filter (EnKF) which may be thought of as a Monte Carlo implementation of the Kalman filter for nonlinear estimation problems. The theory and formulation of the RSEnKF are presented in this paper. The proposed method is superior to the extended risk sensitive filter (ERSF) and the quadrature based risk sensitive filters in terms of estimation accuracy, and is faster than the risk sensitive particle filter (RSPF).

Key Words: Risk sensitive estimation, ensemble Kalman filter, nonlinear estimation, robust filtering.

I. INTRODUCTION

The risk sensitive estimator [1–4] which was originated to minimize the exponential cost function, is believed to have increased robustness compared with risk neutral filters. For linear Gaussian signal models, a closed form solution exists and that has been formulated as the Kalman filter like recursion [1,4]. However, in nonlinear systems the integrals become intractable leading to unavailability of any closed form solution. Initially the problem was solved using the extended Kalman filter based approach [2]. To circumvent the limitations associated with the extended risk sensitive filter (ERSF), quadrature based nonlinear risk sensitive filtering techniques, such as the risk sensitive unscented Kalman filter (RSUKF) [5], and the central difference risk sensitive filter (CDRSF) [6] have been proposed. There is a close connection between the H^∞ error bound and bound for the risk sensitive estimation [1,7] by virtue of which it enjoys greater robustness. A robust unscented Kalman filter [8] has also been formulated by solving the H^∞ filter Riccati equation using the unscented transform. The CDRSF and the RSUKF are computationally efficient, however, they approximate posterior and prior probability density functions to be Gaussian.

To obtain the risk sensitive estimate with desired accuracy for nonlinear non-Gaussian systems, the intractable integrals are solved numerically using points in state space (known as particles) and associated weights. The method is known as the risk sensitive particle filter (RSPF) [9]. Although particle implementation of risk sensitive filtering does not rely on any linear or Gaussian assumption and can be used for any model, for very high dimensional systems the use of RSPF is restricted due to very high computational cost and suffers from the curse of dimensionality problem.

To estimate the states of a very high dimensional system, Evensen [10,11] proposed a nonlinear estimator named as ensemble Kalman filter (EnKF) in the context of weather forecasting [12]. The estimator which is very popular with meteorologists, is able to produce accurate, and computationally efficient estimation, even for a system with very high dimensions.The EnKF [13] is a kind of numerical Monte Carlo (MC) simulation method, used to solve the intractable integrals. In the EnKF, an ensemble of adequate size is formed with the states sampled from the posterior probability density function and measurements. The ensemble is propagated with the help of the state and the measurement equations and updated with the Kalman filter scheme.

In this paper, an alternative approach has been developed for the robust estimation of states using the ensemble Kalman filter. The developed method is based on risk sensitive cost function and is named the risk sensitive ensemble Kalman filter (RSEnKF). The proposed robust filter has been applied to a severely nonlinear single dimensional problem and ballistic target tracking problem. The simulation results obtained from Monte Carlo runs have been provided and compared with its risk neutral counterparts, and the ERSF in terms of estimation accuracy and computational efficiency. The ensemble implementation of the risk sensitive filter could be useful for accurate estimation of very high dimensional systems and significantly broadening the range of practical applications of the risk sensitive filters.

II. RISK SENSITIVE FILTERING

Consider the nonlinear plant described by the process and measurement equations as follows:

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$$x_{k+1} = \phi(x_k) + w_k \quad (1)$$

$$y_k = \gamma(x_k) + v_k \quad (2)$$

Where $x_k \in R^n$ denotes the state of the system, $y_k \in R^p$ is the measurement at the instance k where $k = 0, 1, 2, 3, \ldots, N$, $\phi(x_k)$ and $\gamma(x_k)$ are known nonlinear functions of x_k and k. The process noise $w_k \in R^n$ and measurement noise $v_k \in R^p$ are assumed to be uncorrelated and normally distributed with covariance Q_k and R_k, respectively. The following notations have been used to represent the probability density functions:

$$f(x_{k+1}|x_k) \triangleq p_{x_{k+1}|x_k}(.|x_k)$$

and

$$g(y_k|x_k) \triangleq p_{y_k|x_k}(.|x_k).$$

The objective is to estimate a known function $\Phi(x)$ of the state variables. The estimate is designated as $\hat{\Phi}(x)$ and its optimal value in the risk sensitive sense is denoted as $\hat{\Phi}^*(x)$ which minimizes the cost function [1]:

$$C(\hat{\Phi}_1, \ldots, \hat{\Phi}_k) = E\left[\exp\left(\mu_1 \sum_{i=1}^{k-1} \rho_1(\Phi(x_i) - \hat{\Phi}_i^*)\right) + (\mu_2 \rho_2(\Phi(x_k) - \hat{\Phi}_k^*))\right]$$

where $\mu_1 \geq 0$ and $\mu_2 > 0$ are two risk parameters. Functions $\rho_1(.)$ and $\rho_2(.)$ are both strictly convex, continuous and bounded from below, attaining global minima at 0. In particular, the minimum risk sensitive estimate (MRSE) is defined by

$$\hat{\Phi}_k^* = \underset{\hat{\Phi}_k \in R}{arg\,min}\, C(\hat{\Phi}_1^*, \ldots, \hat{\Phi}_{k-1}^*, \hat{\Phi}_k). \quad (3)$$

It can be shown that [14] the MRSE satisfies the following recursion:

$$\sigma_{k+1}(x_{k+1}) = \int f(x_{k+1}|x_k) g(y_k|x_k) \times \exp(\mu_1 \rho_1(\Phi(x_k) - \hat{\Phi}_k^*)) \sigma_k(x_k) dx_k \quad (4)$$

$$\hat{\Phi}_k^* = \underset{\alpha \in R}{arg\,min} \int \exp(\mu_2 \rho_2(\Phi(x_k) - \alpha)) \sigma_k(x_k) dx_k \quad (5)$$

where $\sigma_k(x_k)$ represents an information state [1] which provides information about the system's states [15]. In this formulation, the information state is a probability density function into which a component of cost is included. The information state may be normalized and α is a parameter vector in general. If we assume the variables to be estimated are the state variables themselves ($\Phi(x) = x$), and both the convex functions $\rho_1(.)$ and $\rho_2(.)$ are known quadratic functions of vectors, *i.e.*, $\rho_j(\varepsilon) = \varepsilon^T \varepsilon$ for $j = 1, 2$; for prior state estimation ($\hat{x}_{k|k-1}$), then (4) and (5) can be written as

$$\sigma_{k+1|k}(x_{k+1}) = \int f(x_{k+1}|x_k) g(y_k|x_k) \exp(\mu_1 (x_k - \hat{x}_{k|k-1})^T (x_k - \hat{x}_{k|k-1})) \sigma_{k|k-1}(x_k) dx_k \quad (6)$$

and

$$\hat{x}_{k|k-1} = \underset{\alpha \in R}{arg\,min} \int \exp(\mu_2 (x_k - \alpha)^T (x_k - \alpha)) \sigma_{k|k-1}(x_k) dx_k \quad (7)$$

Similar recursion for posterior estimation is available in [16] and is not mentioned here.

III. FORMULATION OF RISK SENSITIVE ENSEMBLE KALMAN FILTER

To obtain the risk sensitive prior estimation, closed form evaluation of the integral described in (6) is necessary. But the closed form solution only exists for linear Gaussian systems. For the nonlinear systems the integral is intractable. In this paper, an ensemble of state vectors has been used to approximate the probability density function with Gaussian distribution hence calculate the mean and covariance of the distribution. The algorithm formulated in this section is prior in nature. Similarly the posterior form of the RSEnKF can be formulated and is omitted here.

In order to formulate the EnKF with risk sensitive cost function, (6) is decomposed as follows:

$$\sigma_{k+1|k}(x_{k+1}) = \int f(x_{k+1}|x_k) \sigma_{k|k} dx_k \quad (8)$$

where

$$\sigma_{k|k}(x_k) = g(y_k|x_k) \exp(\mu_1 (x_k - \hat{x}_{k|k-1})^T \times (x_k - \hat{x}_{k|k-1})) \sigma_{k|k-1}(x_k). \quad (9)$$

Let us define $\sigma_{k|k-1}^{\dagger}(x_k)$ as

$$\sigma_{k|k-1}^{\dagger} = \exp(\mu_1 (x_k - \hat{x}_{k|k-1})^T (x_k - \hat{x}_{k|k-1})) \sigma_{k|k-1}(x_k) \quad (10)$$

Then (9) can be written as

$$\sigma_{k|k}(x_k) = g(y_k|x_k) \sigma_{k|k-1}^{\dagger}(x_k) \quad (11)$$

The information state, $\sigma_k(x_k)$, with the normalization may be interpreted as a probability density function. Equation (8) may be considered as the time update step in risk sensitive estimation. The optimal estimate of state is determined using the equation (7). The expression (11) defines the risk update which is unique for risk sensitive estimation. In the EnKF, the ensemble of adequate size is formed with the states sampled from the posterior probability density function and measurements. The ensemble is propagated with the help of state and measurement equations and updated with the Kalman filter scheme.

3.1 RSEnKF algorithm

Step 1 Filter initialization

- Initialize the filter with $\bar{\sigma}_{0|0} = \hat{x}_{0|0}$ and covariance $P_{0|0}$.

- Augment the state vector with measurement [13] and draw the samples from initial distribution to form initial ensemble of size N.

$$\chi_0 = \begin{bmatrix} X_0 \\ Y_0 \end{bmatrix} = \begin{bmatrix} \sigma_0^1 & \sigma_0^2 & \sigma_0^3 & \dots & \sigma_0^N \\ y_0^1 & y_0^2 & y_0^3 & \dots & y_0^N \end{bmatrix}$$

where $\chi_0 \in R^{(n+p)\times N}$, $X_0 \in R^{n\times N}$, $Y_0 \in R^{p\times N}$, and $\sigma_0^j \sim \aleph(\bar{\sigma}_{0|0}, P_{0|0})$. y_0^j are obtained from $\gamma(\sigma_0^j)$.

Step 2 Predictor step

- Propagate the ensemble members

$$\sigma_{k+1|k}^j = \phi(\sigma_{k|k}^j) + w_k$$

$$y_{k+1|k}^j = \gamma(\sigma_{k+1|k}^j)$$

- Evaluate time updated ensemble mean

$$\hat{\chi}_{k+1|k} = \frac{1}{N}\sum_{j=1}^{N} \chi_{k+1|k}^j$$

- Evaluate the ensemble error covariance

$$P_{k+1|k} = L_{k+1|k} L_{k+1|k}^T$$

where

$$L_{k+1|k} = \frac{1}{\sqrt{N-1}}[(\chi_{k+1|k}^1 - \hat{\chi}_{k+1|k})(\chi_{k+1|k}^2 - \hat{\chi}_{k+1|k}) \dots (\chi_{k+1|k}^N - \hat{\chi}_{k+1|k})]$$

Step 3 Optimal risk sensitive estimate

- Calculate mean of the state ensemble

$$\bar{\sigma}_{k+1|k} = G\hat{\chi}_{k+1|k}$$

Where G is a matrix given by $G = [I_{n\times n}\ 0_{n\times p}]$

- With Gaussian assumption, the optimal state estimation is

$$\hat{x}_{k+1|k} = \bar{\sigma}_{k+1|k}$$

Step 4 Risk sensitive update

- The mean remains the same and the covariance is updated with

$$P_{k+1|k}^{\dagger} = (P_{k+1|k}^{-1} - 2\mu_1 I)^{-1}$$

Step 5 Corrector step or measurement update

- Calculate the Kalman gain matrix

$$K_{k+1} = P_{k+1|k}^{\dagger} H^T (H P_{k+1|k}^{\dagger} H^T + R_k)^{-1}$$

where H matrix is given as $H = [0_{p\times n}\ I_{p\times p}]$.

- Update the ensemble points

$$\chi_{k+1|k+1}^j = \chi_{k+1|k}^j + K_{k+1}(y_{k,o}^j - y_{k+1|k}^j)$$

where, $y_{k,o}^j$ are obtained from the distribution of measurement noise, $y_{k,o}^j \sim \aleph(y_k, R_k)$.

Remarks.

- For a nonlinear process and linear measurement system, the ensemble can be formed with the states only. Correspondingly, the update equation of ensemble needs little modification.
- In should be noted that the ensemble variance is calculated by dividing with N-1 instead of N, because the population mean is unknown. The correction is known as Bessel's correction. It reduces the bias in the estimation of the population variance [17].
- The RSEnKF is computationally advantageous compared to other numerical filtering techniques. Often it is argued that for a large dimensional problem, an ensemble size of 50–100 is sufficient to render a useful estimate [13]. However, the statement should not be over emphasized. For a large dimensional highly nonlinear system and measurement, more than 100 ensembles may be necessary to avoid the divergence. It should also be noted that with the increased dimension, the number of row of an ensemble matrix increases thus increasing computation time.
- The risk sensitive parameter μ_1 should be chosen low enough such that during simulation the condition $P_{k+1|k}^{\dagger} > 0$ is satisfied.
- From Step (4) it is clear that the risk sensitive updated covariance is larger than the risk neutral prior error covariance. This is known as covariance inflation in the EnKF community [18]. In [19] it has been shown that the ensemble implementation of the H^{∞} filter is equivalent to an EnKF equipped with a specific covariance inflation technique. The same statement is true for the risk sensitive implementation of the EnKF filter.
- To execute the risk sensitive update step (Step (4)) for a large dimensional system a large matrix needs to be inverted. The Computational cost to invert a large matrix straight forwardly is high and it may not appear practical to do. A more cost-effective strategy can be to do it in the framework of the singular evolutive interpolation Kalman (SEIK) filter [20,21], in which one only inverts a matrix in the dimension of the ensemble size minus one. Such implementations are reported in recent works [22,23].

IV. CASE STUDY

4.1 Example 1

This example uses a plant where both the state model and the measurement model are severely nonlinear. The plant model, inspired by [24] has strong nonlinearity, with one unstable and two stable equilibrium points. A brief description of the plant is provided below. The process and measurement equations are given by

$$x_{k+1} = \phi(x_k) + w_k$$

where $\phi(x) = x + \Delta t 5x(1 - x^2)$, and $w_k \sim \aleph(0, b^2\Delta t)$, and

$$y_k = \gamma(x_k) + v_k$$

where $\gamma(x) = \Delta t x(1 - 0.5x)$, $v_k \sim \aleph(0, d^2\Delta t)$, the value of $\Delta t = 0.01$sec, $x_0 = -0.2$, $\hat{x}_{0|0} = 0.8$, $P_{0|0} = 2$, $b = 0.5$, and $d = 0.1$.

The system has three equilibrium points, of which, the one at the origin is unstable and the other two are at ±1 and stable. In the absence of any bias, the system hovers around either of the two stable equilibrium points. The problem becomes challenging because moderate estimation error forces the estimate to settle down at the wrong equilibrium point, leading to a track loss situation.

Simulation Results: Robustness and bandwidth of the risk filter is increased by enhancing the value of μ_1 and therefore is desirable. However there is a limit beyond which one cannot increase the risk sensitive parameter. The risk sensitive parameter is chosen such that it satisfies the condition $(P_k^{-1} - 2\mu_1 I) > 0$ for each step over the time horizon. In other words, $\mu_1 \leq 1/(2\|P\|)$, where $\|P\|$ is the largest eigenvalue of P, taken all over k. To select the value of μ_1, the P matrix can be guessed from a knowledge of physics or from a run of EKF, and the highest permissible value of μ_1 is deduced from simulation. Experimental evaluations of estimators' accuracy are performed with different risk-sensitive parameters (lower than the highest permissible value) and finally the value of risk sensitive parameter is frozen. The same pragmatic is used to simulate all the risk sensitive filters.

The risk sensitive parameter (μ_1) has been chosen as 0.0756 during simulation. Truth value and estimated state obtained from the extended risk sensitive filter (ERSF), the risk sensitive unscented Kalman filter (RSUKF), the central difference risk sensitive filter (CDRSF), the risk sensitive particle filter (RSPF) with 1000 particles and proposed risk sensitive ensemble Kalman filter (RSEnKF) with ensemble size of 100 have been plotted in Fig. 1 for a single representative run. It has been observed that the ERSF looses track where all other risk sensitive filters settle at proper equilibrium point.

The root mean square error (RMSE) has been carried out for 1000 Monte Carlo runs for all the filters and shown in Fig. 2. Fig. 2 reveals that the RMSE of the ERSF diverges (due to large population of track loss cases), whereas the RMSEs of the CDRSF and the RSUKF settle down approximately at 0.45 and 0.22, respectively, after 1 second. The RMSEs of the RSPF and the RSEnKF converge at 0.17 and 0.15, respectively.

The performance of the filters is compared in terms of percentage of fail count. Percentage of fail count is defined as the number of track loss cases out of 100 (in a population of 1000 Monte Carlo runs) runs. When a filter fails to track the truth, it settles at the wrong equilibrium point. The percentage of fail count for different risk sensitive filters has been tabulated in Table I. From Fig. 2 and Table I it is claimed that the proposed RSEnKF is better than the ERSF, the CDRSF and the RSUKF. The

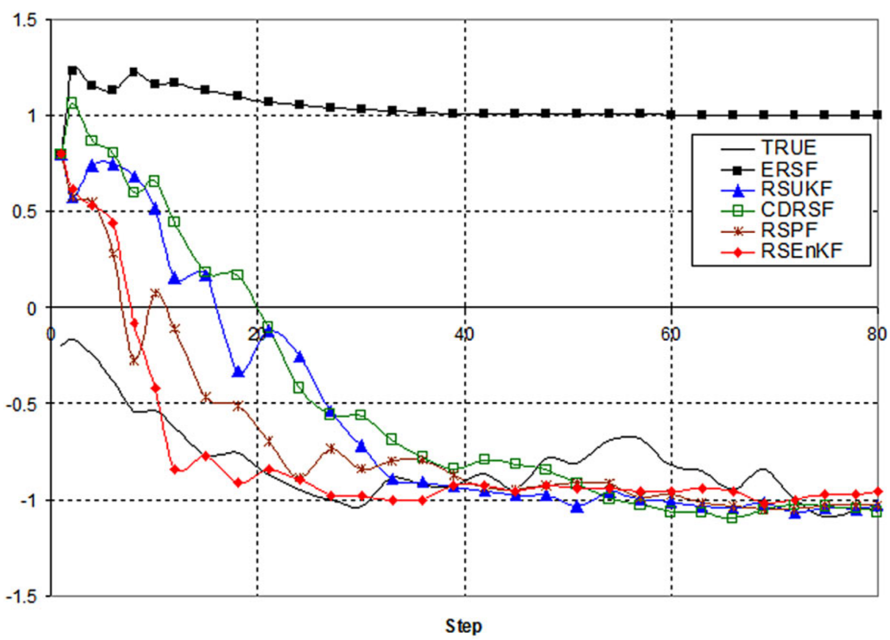

Fig. 1. Truth and estimated values for single representative run.

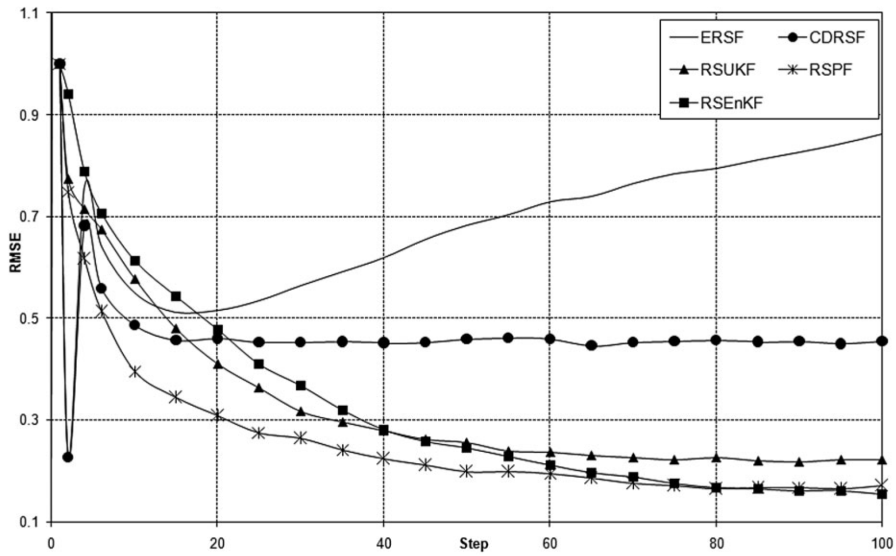

Fig. 2. RMSE for 1000 MC run.

Table I. Percentage fail count.

Filter	Fail count in %
ERSF	26
CDRSF	4.8
RSUKF	1
RSPF	0
RSEnKF	0

Table II. Time required for single run.

Filter	Run time (sec)
ERSF	0.001
CDRSF	0.002
RSUKF	0.011
RSPF-1000	10.760
RSEnKF-100	0.384

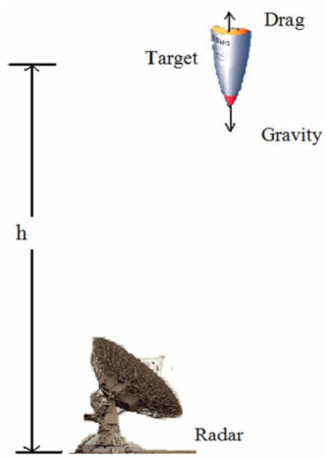

Fig. 3. Ballistic target tracking scenario using ground radar.

performance of the RSEnKF is comparable to the RSPF in terms of estimation accuracy.

The risk filters are also compared in terms of computational efficiency. Averaged time required to run the filters in MATLAB software are observed and summarize in Table II. From the table it can be observed that the computational efficiency of the RSEnKF is far better than the RSPF.

4.2 Example 2

Tracking of a ballistic target using ground radar measurement in the re-entry phase is considered here. When the ballistic target re-enters in the atmosphere, the speed of the target is very high and the time to make contact with the ground is very small allowing small little for estimator convergence.

In this paper, the target is assumed to be falling vertically downward as shown in Fig. 3. So the motion of the target becomes single dimensional in nature. I consider the case where the ballistic coefficient (β), which is a function of mass, and shape, is unknown. The main objective is to estimate the position, velocity, and ballistic coefficient of the target with the help of ground radar measurements. I adopt the state space formulation of the problem as described in [25,26]. The motion of the target has been evaluated with the assumption that only the drag and the gravity are the two forces acting on it (neglecting lift and all other forces). The gravity is also considered to be constant with respect to altitude. Under the above described assumptions, the kinematics of the target is governed by the following continuous time domain equations.

$$\dot{h} = -\upsilon \quad (12)$$

$$\dot{\upsilon} = -\frac{\rho(h) g \upsilon^2}{2\beta} + g \quad (13)$$

$$\dot{\beta} = 0 \quad (14)$$

Where, h is the altitude (position) in meters, v is the velocity in m/s, $\rho(h)$ is the air density, g is the acceleration due to gravity (9.81 m/s^2) and β is the ballistic coefficient. Air density is an exponential function of altitude and is given by $\rho(h) = \gamma e^{-\eta h}$. Where, $\gamma = 1.754$, and $\eta = 1.49 \times 10^{-4}$. Now, to implement the state estimator, the target dynamics are discretised. The discretised state space model is given by

$$x_{k+1} = f(x_k) + w_k \quad (15)$$

where $f(x_k)$ is a nonlinear function used to describe the state evolution and can be expressed as

$$f(x_k) = \phi x_k - G[D(x_k) - g] \quad (16)$$

where x_k, ϕ and G are given in terms of sampling time τ as $x_k = [h\ v\ \beta]^T$, $\phi = [1\ -\tau\ 0;\ 0\ 1\ 0;\ 0\ 0\ 1]$, $G = [0\ \tau\ 0]^T$. The drag is expressed as

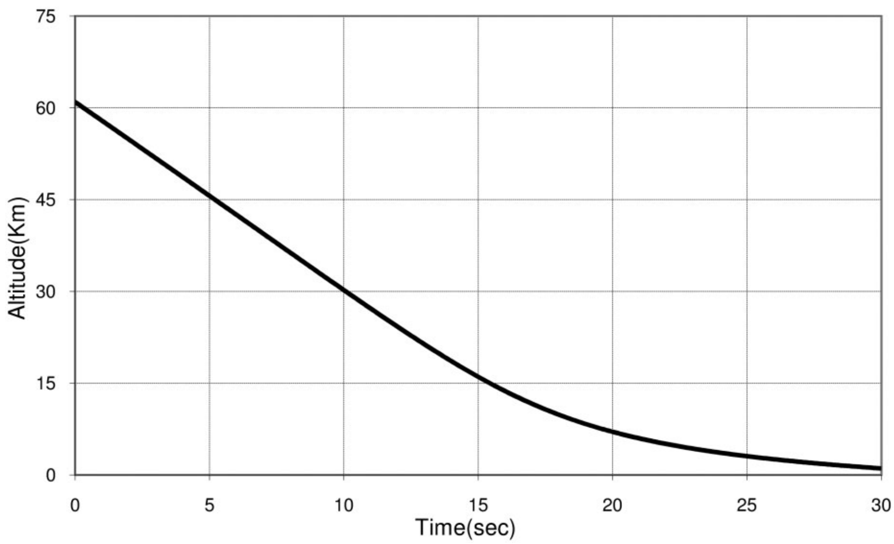

Fig. 4. Typical target trajectory—altitude vs time.

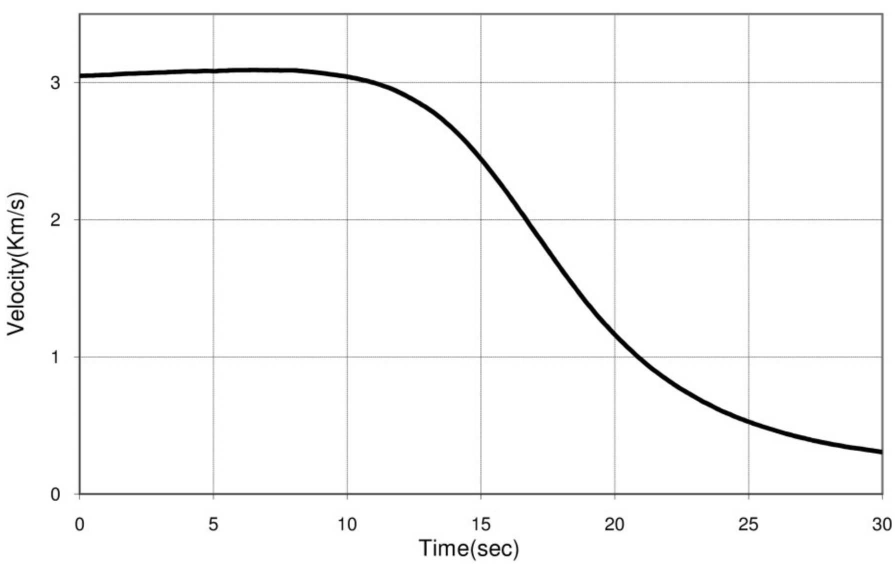

Fig. 5. Typical target trajectory—velocity vs time.

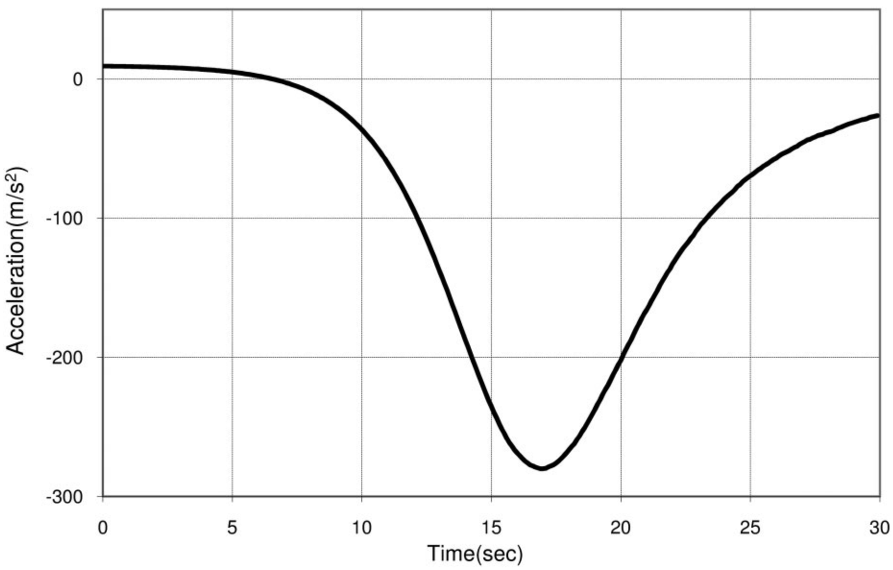

Fig. 6. Typical target trajectory—acceleration vs time.

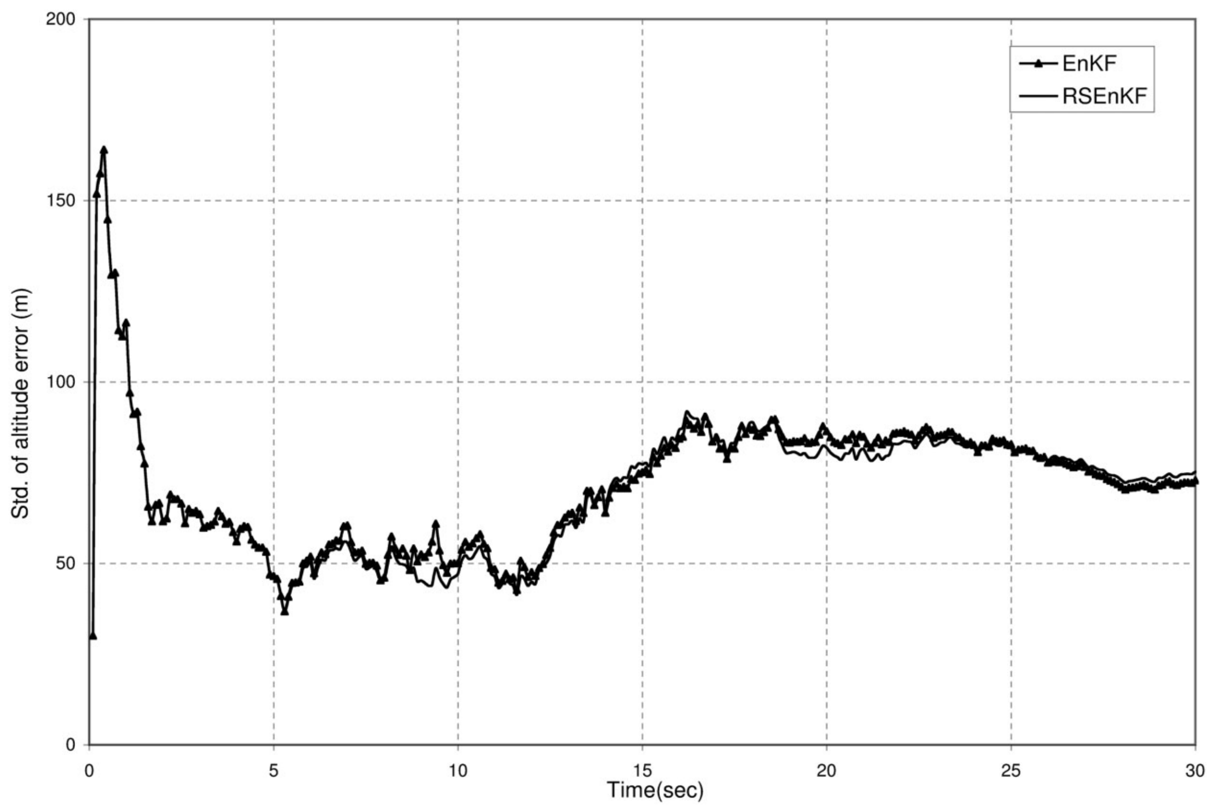

Fig. 7. Std. of position estimation error obtained from 50 MC runs.

$$D(h_k, v_k, \beta_k) = \frac{g.\rho(h_k).v_k^2}{2\beta_k} \tag{17}$$

It is to be noted that the nonlinearity in target dynamics arises due to drag only. w_k is process noise and arises due to imperfection in kinematics model. The process noise is assumed to be Gaussian, and is characterized by zero mean Q_k covariance, $Q = \left[q_1 \frac{\tau^3}{3} \quad q_1 \frac{\tau^2}{2} \quad 0; q_1 \frac{\tau^2}{2} \quad q_1\tau \quad 0; 0 \quad 0 \quad q_2\tau \right]$. Where q_1 in (m^2/s^3) and q_2 in $(kg^2/m^{-2}\, s^{-5})$ are the tuning parameters to be

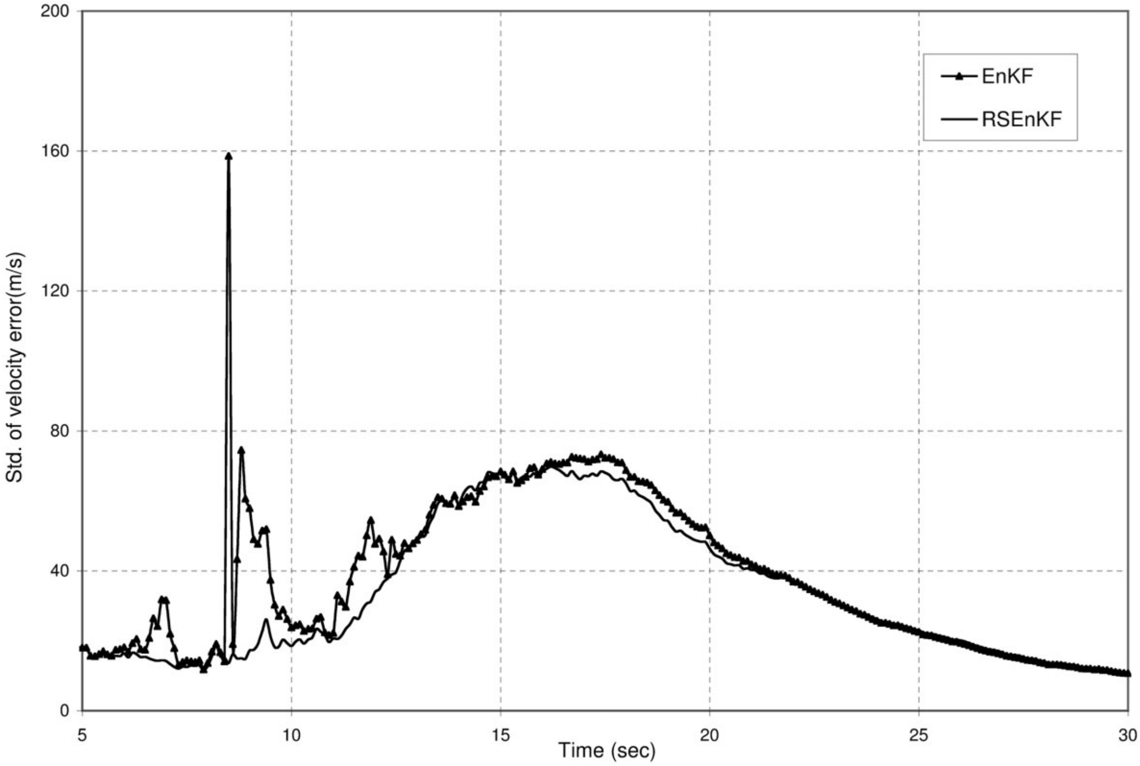

Fig. 8. Std. of velocity estimation error obtained from 50 MC runs.

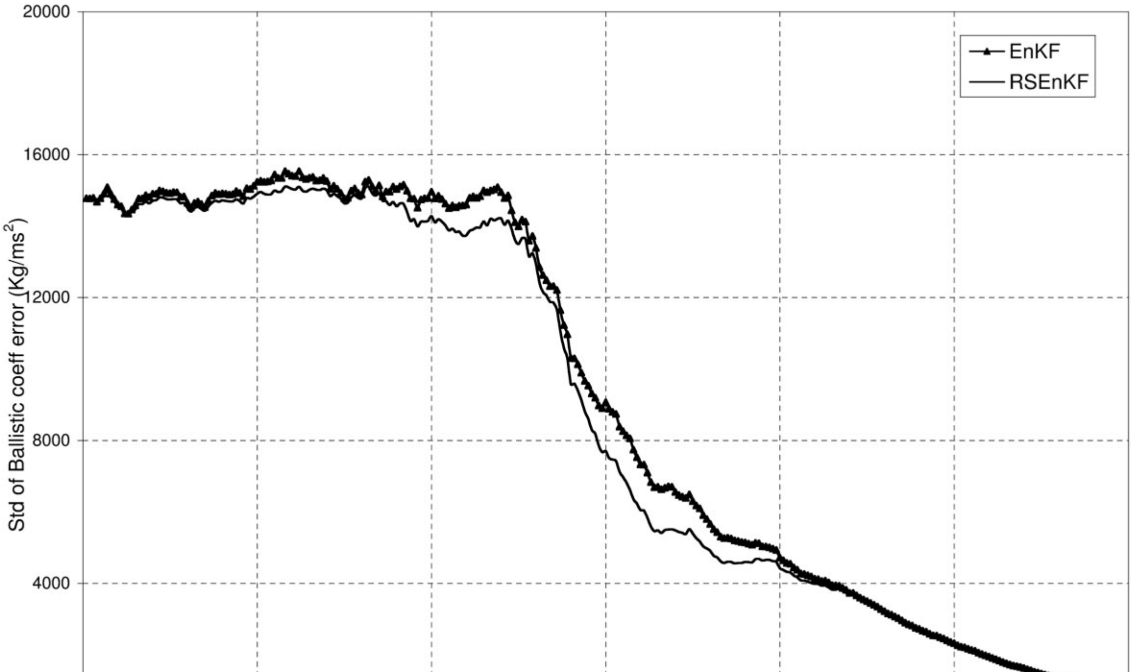

Fig. 9. Std. of ballistic coefficient estimation error obtained from 50 MC runs.

selected by designers to model the process noise in target dynamics.

The radar located on the ground is assumed to measure the altitude of the target at any instance of time. The measurement equation is assumed to be linear and corrupted with white Gaussian noise:

$$y_k = Hx_k + v_k \tag{18}$$

where $H = [1\ \ 0\ \ 0]$, and v_k is zero mean white Gaussian measurement noise with variance R_k.

A typical target's altitude, velocity and acceleration without any process noise are plotted in Fig. 4 to Fig. 6, respectively. The truth trajectory of the ballistic target is simulated with the initial height and velocity as $h = 60960$ m, and $v = 3048$ m/s, respectively. The ballistic coefficient (β) is initialized from the beta distribution with both the parameters 1.1, *i.e.* $\beta \sim Be(1.1, 1.1)$, with upper and lower level boundaries as 10,000 kg/ms^2 and 63,000 kg/ms^2 respectively. Ballistic coefficient is initialized randomly as there is no clue available about the shape and size of the ballistic target to be tracked.

The filter or estimator is initialized with the random number generated from Gaussian distribution with mean and covariance as $\hat{x}_{0|0} = [60960\quad 3048\quad mean(\beta_0)]^T$, and $P(X)_{0|0} = \left[R_k\quad \frac{R_k}{\tau}\quad 0; \frac{R_k}{\tau}\quad \frac{R_k}{\tau^2}\quad 0; 0\quad 0\quad \sigma_\beta^2\right]$, β_0 and σ_β are the mean and standard deviation of the random number generated from beta distribution, described above.

The above described ballistic target tracking problem has been solved using RSEnKF and EnKF. The sampling time τ is taken as 0.1 and simulation is performed for 30 s duration. The process noise for truth as well as filter is taken with $q_1 = q_2 = 5$. The measurement noise covariance (R_k) is taken as $R_k = 200^2$. In filter η has been taken as 1.49×10^{-4} where as in truth the value of η is taken as $(1.49 + 0.25) \times 10^{-4}$. The simulation is carried out with 50 ensemble and results are compared for 50 Monte Carlo runs.

The standard deviation of the altitude, velocity, and ballistic coefficient obtained from the EnKF and the RSEnKF (in presence of mismatch of η) are plotted in Fig.7 to Fig. 9, respectively. From Fig. 7, it has been observed that the standard deviation of the position errors obtained from the RSEnKF is almost same as the EnKF. Fig. 8 and Fig. 9 reveal that the standard deviations of velocity and ballistic coefficient estimation errors are lower in RSEnkF compared to ordinary EnKF. Hence, it could be concluded that the RSEnKF tracks better compared to the EnKF in presence of model parameter mismatch.

V. DISCUSSION AND CONCLUSION

In this paper a solution for the nonlinear risk sensitive estimation problem has been proposed in the ensemble Kalman filter framework. The theory and algorithm of the RSEnKF have been illustrated. The proposed method has been applied to a single dimensional severely nonlinear problem as well as ballistic target tracking on re-entry problem. The proposed filter has been found to be more accurate compared to the ERSF. The computational efficiency of the proposed RSEnKF is much better than the RSPF. Due to the parallel processing nature of the algorithm, absence of the curse of dimensionality problem, and computational efficiency compared to RSPF, the proposed filter may find a place for robust estimation of large dimensional system where the application of the RSPF would be inappropriate due to it's computational inefficiency. The implementation and detailed comparison of the proposed filter with the RSPF for very large dimensional system remain under the scope of future works.

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